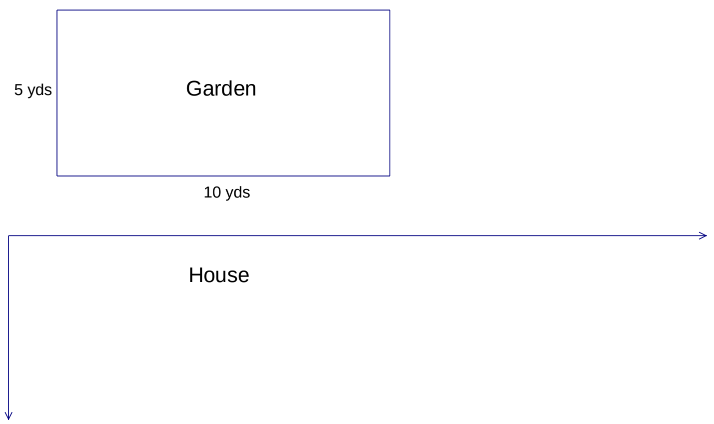
Around the Garden Learning Task (Domain and Range in Context)

1. Claire has decided to plant a rectangular garden in her back yard using 30 pieces of fencing that were given to her by a friend. Each piece of fencing is a panel 1 yard wide and high enough to keep deer out. She wants to determine the possible dimensions of her garden, assuming that she uses all of the fencing and does not cut the panels. She decides to let x be the garden dimension parallel to the back of her house, and to let y be the other dimension. For example, she begins by placing ten panels (10 yards) parallel to the back side of her house and realizes that the other dimension of her garden then will have to be 5 yards, as shown in the picture below. She summarizes this result as follows: When $x = 10$, $y = 5$.



   a. Explain why y must be 5 when x is 10.
   b. Make a table showing the possible lengths and widths for the garden.
   c. If $x = 15$, what could y be? Explain why Claire would be unlikely to build such a garden.
   d. Can x be 16? What is the maximum possible width? Explain.
   e. Write a formula relating the width and length of the garden. For what possible lengths and widths is this formula appropriate? Express these values mathematically.
   f. Find the perimeters of each of these possible gardens. What do you notice? Explain why this happens.
   g. Make a graph of the possible dimensions of Claire's garden.
   h. What would it mean to connect the dots on your graph? Does connecting the dots make sense for this context? Explain.
   i. As the x-dimension of the garden increases by 1 yard, what happens to the y-dimension? Does it matter what x-value you start with? How do you see this in the graph? In the table? In your formula? What makes the dimensions change together in this way?

2. After listing the possible rectangular dimensions of the garden, Claire realizes that she needs to pay attention to the area of the garden, because that is what determines how many plants she will be able to grow.
   a. Will the area of the garden change as the *x*-dimension changes? Make a prediction, and explain your thinking.
   b. Make a table showing all the possible *x*-dimensions for the garden and the corresponding areas. (To facilitate your calculations, you might want to include the *y*-dimensions in your table.)
   c. Make a graph showing the relationship between the *x*-dimension and the area of the garden. Should you connect the dots? Explain.
   d. Write a formula showing how to compute the area of the garden, given its *x*-dimension.

3. Because the area of Claire's garden depends upon the *x*-dimension, we can say that the area is a function of the *x*-dimension. Each *x*-dimension is an input value for the area function, and the resulting area is the corresponding output value. The set of all possible input values for a function is called the *domain* of the function. The set of all possible output values is called the *range* of the function.
   a. What is the domain of the area function for this context?
   b. How is the question about domain related to the question about connecting the dots on the graph?
   c. What is the maximum area of the garden, and what are its dimensions? How do you see this in your table? In your graph?
   d. What is the minimum area of the garden, and what are its dimensions? How do you see this in your table? In your graph?
   e. What is the range of the area function for this context? How can you see the range in your table? In your graph?
   f. Claire's neighbor Javier noticed that the graph is symmetric. Describe the symmetry, specifically indicating the line of symmetry. What about the context causes this symmetry?
   g. As the *x*-dimension of the garden increases by 1 yard, what happens to the area? Does it matter what *x*-dimension you start with? How do you see this in the graph? In the table? Explain what you notice.

4. Later that summer, Claire's sister-in-law Kenya mentions that she wants to use 30 yards of chain-link fence to build a pen for her pet rabbits. Claire experiences déjà vu and shares the solution to her garden problem but then she realizes that Kenya's problem is slightly different.
   a. In this case, Claire and Kenya agree that the *x*-dimension does not have to be a whole number. Explain.
   b. Despite the fact that the rabbit pen would not be useful with an *x*-dimension of 0, it is often valuable, mathematically, to include such "limiting cases," when possible. Why would a rabbit pen with an *x*-dimension of 0 be called a limiting case in this situation? Why would the shape of the garden be called a "degenerate rectangle"?
   c. Are there other limiting cases to consider in this situation? Explain.
   d. Make a table for the area versus the *x*-dimension of Kenya's garden. Be sure to include some non-whole-number *x*-dimensions.
   e. Make a graph of the area versus the *x*-dimension of Kenya's garden. (Note, if the limiting case is *not included*, the point is plotted in the graph as a small "open circle." If the limiting case is *included* the point is plotted in the graph as a small "closed circle.")
   f. Write a formula showing how to compute the area of the rabbit pen, given its *x*-dimension.

5. Kenya uses the table, graph, and formula to answer questions about the possibilities for her rabbit pen.
   a. Estimate the area of a rabbit pen of with an *x*-dimension of 10 feet (not yards). Explain your reasoning.
   b. Estimate the *x*-dimension of a rectangle with an area of 30 square yards. Explain your reasoning.
   c. What is the domain of the function relating the area of the rabbit pen to its *x*-dimension. How do you see the domain in the graph?
   d. What is the area and what are the dimensions of the pen with maximum area? Explain. And what do you notice about the shape of the pen?
   e. What is the range of the area function for the rabbit pen? How can you see the range in the graph?
   f. As the *x*-dimension of Kenya's garden increases, sometimes the area increases and sometimes the area decreases. For what *x*-values does the area increase as *x* increases? For what *x*-values does the area decrease as *x* increases? [Note to teachers: Do not use interval notation.]

When using tables and formulas we often look at a function a point or two at a time, but in high school mathematics, it is important to begin to think about "the whole function," which is to say all of the input-output pairs. A graph of a function is very useful for considering questions about "whole functions," but keep in mind that a graph might not show all possible input-output pairs.

Two functions are equal (as whole functions) if they have exactly the same input-output pairs. In other words, two functions are equal if they have the same domain and if the output values are the same *for each input value in the domain*. From a graphical perspective, two functions are equal if their graphs have exactly the same points.

6. Kenya's rabbit pen and Claire's garden are very similar in some respects but different in others. These two situations involve different *functions*, even though the *formulas* are the same.
   a. If Kenya makes the pen with maximum area, how much more area will her rabbit pen have than Claire's garden of maximum area? How much area is that in square feet?
   b. What could Claire have done to have built her garden with the same area as the maximum area for Kenya's rabbit pen? Do you think this would have been worthwhile?
   c. Describe the similarities and differences between Kenya's rabbit pen problem and Claire's garden problem. Consider the tables, the graphs, the formulas, and the problem situations. Use the words *domain* and *range* in your response.