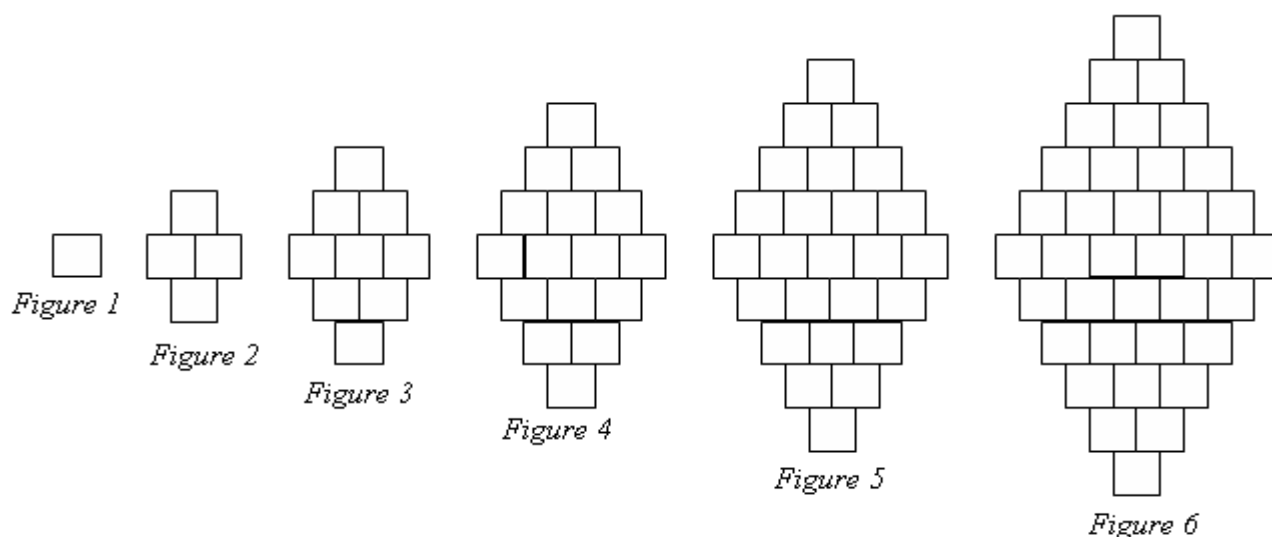


## Tiling Learning Task (Exploring Algebraic Expressions)

Latasha and Mario are high school juniors who worked as counselors at a day camp last summer. One of the art projects for the campers involved making designs from colored one-square-inch tiles. As the students worked enthusiastically making their designs, Mario noticed one student making a diamond-shaped design and wondered how big a design, with the same pattern, that could be made if all 5000 tiles available were used. Later in the afternoon, as he and Latasha were putting away materials after the children had left, he mentioned the idea to Latasha. She replied that she saw an interesting design too and wondered if he were talking about the same design. At this point, they stopped cleaning up and got out the tiles to show each other the designs they had in mind.

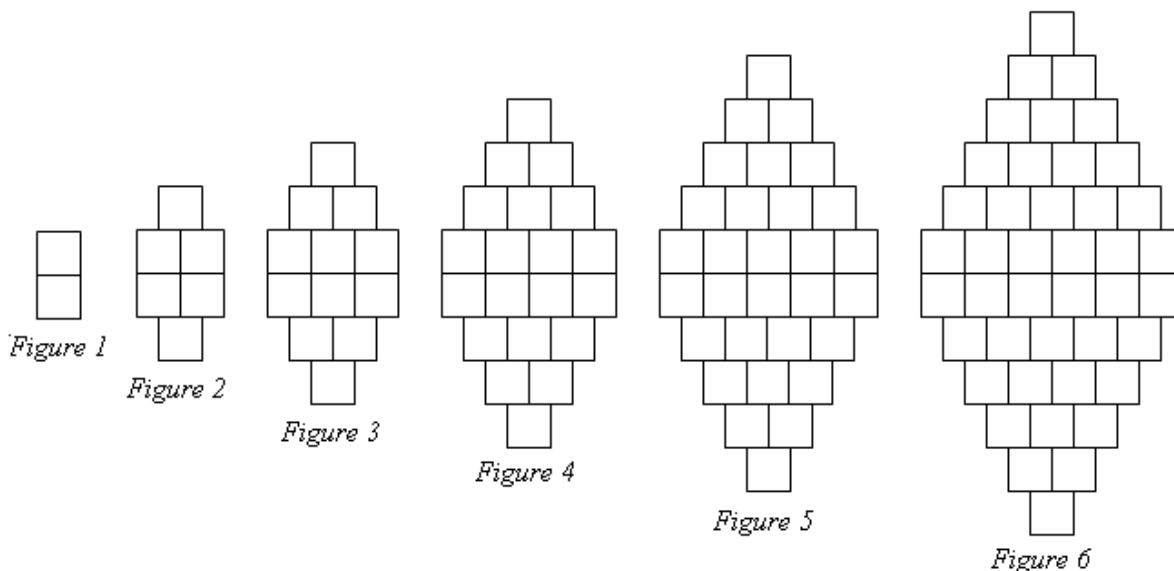
Mario presented the design that interested him as a sequence of figures as follows:



1. To make sure that you understand the design that was of interest to Mario, answer the following questions.
  - a) How many rows of tiles are in each of Mario's figures?
  - b) What pattern do you observe that relates the number of rows to the figure number? Explain in a sentence.
  - c) Use this pattern to predict the number rows in Figure 12, Figure 47, and Figure 111 if these figures were to be drawn.
  - d) Write an algebraic expression for the number of rows in Figure  $k$ . Explain why your pattern will always give the correct number of rows in Figure  $k$ . Can your expression be simplified? If so, simplify it.
  - e) What is the total number of tiles in each figure above?
  - f) What pattern do you observe that relates the total number of tiles to the figure number? Explain in a sentence.
  - g) Use this pattern to predict the total number of tiles in Figure 12, Figure 47, and Figure 111 if these figures were to be drawn.
  - h) Write an algebraic expression for the total number of tiles in Figure  $k$ . Explain why your pattern will always give the correct total number of tiles in Figure  $k$ .

- i) Write a summary of your thoughts and conclusions regarding Mario's sequence of figures.

When Latasha saw Mario's figures, she realized that the pattern Mario had in mind was very similar to the one that caught her eye, but not quite the same. Latasha pushed each of Mario's designs apart and added some tiles in the middle to make the following sequence of figures.



2. Answer the following questions for Latasha's figures.
- How many rows of tiles are in each of the figures above?
  - What pattern do you observe that relates the number of rows to the figure number? Explain in a sentence.
  - Use this pattern to predict the number rows in Figure 12, Figure 47, and Figure 111 if these figures were to be drawn.
  - Write an algebraic expression for the number of rows in Figure  $k$ . Explain why your pattern will always give the correct number of rows in Figure  $k$ . Can your expression be simplified? If so, simplify it.
  - What is the total number of tiles in each figure above?
  - What pattern do you observe that relates the total number of tiles to the figure number? Explain in a sentence.
  - Use this pattern to predict the total number of tiles in Figure 12, Figure 47, and Figure 111 if these figures were to be drawn.
  - Write an algebraic expression for the total number of tiles in Figure  $k$ . Explain why your pattern will always give the correct total number of tiles in Figure  $k$ .
  - Give a geometric reason why the number of tiles in Figure  $k$  is always an even number.
  - Look at the algebraic expression you wrote in part h. Give an algebraic explanation of why this expression always gives an even number. [Hint: If your expression is not a product, use the distributive property to rewrite it as a product.]
  - Write a summary of your thoughts and conclusions regarding Mario's sequence of figures.

3. Mario started the discussion with Latasha wondering whether he could make a version of the diamond pattern that interested him that would use all 5000 tiles that they had in the art supplies. What do you think? Explain your thoughts. If you can use all 5000 tiles, how many rows will the design have? If a similar design cannot be made, what is the largest design that can be made with the 5000 tiles, that is, how many rows will this design have and how many tiles will be used? Explain your thoughts.
4. What is the largest design in the pattern Latasha liked that can be made with no more than 5000 tiles? How many rows does it have? Does it use all 5000 tiles? Justify your answers.
5. Let  $M_1, M_2, M_3, M_4,$  and so forth represent the sequence of numbers that give the total number of tiles in Mario's sequence of figures.

Let  $L_1, L_2, L_3, L_4$  and so forth represent the sequence of numbers that give the total number of tiles in Latasha's sequence of figures.

Write an equation that expresses each of the following:

- a) the relationship between  $L_1$  and  $M_1$
- b) the relationship between  $L_2$  and  $M_2$
- c) the relationship between  $L_3$  and  $M_3$
- d) the relationship between  $L_4$  and  $M_4$
- e) the general relationship between  $L_k$  and  $M_k$ , where  $k$  can represent any positive integer.