Georgia Standards of Excellence Curriculum Frameworks

Mathematics

GSE Grade 7

Unit 3: Ratio and Proportional Relationships

Georgia Department of Education

Richard Woods, Georgia's School Superintendent

"Educating Georgia's Future"

Unit 3

Ratios and Proportional Relationships

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OVERVIEW

The units in this instructional framework emphasize key standards that assist students to develop a deeper understanding of numbers. They learn to express different representations of rational numbers (e.g., fractions, decimals, and percent's), discover how to identify and explain the constant of proportionality, and represent proportional relationships and scale drawings within real-world contexts. The Big Ideas that are expressed in this unit are integrated with such routine topics as estimation, mental and basic computation. All of these concepts need to be reviewed throughout the year.

Take what you need from the tasks and modify as required. These tasks are suggestions, something that you can use as a resource for your classroom.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students make sense of ratio and unit rates in real-world contexts. They persevere by selecting and using appropriate representations for the given contexts.

2. Reason abstractly and quantitatively. Students will reason about the value of the rational number in relation the models that are created to represent them.

3. Construct viable arguments and critique the reasoning of others. Students use arguments to justify their reasoning when creating and solving proportions used in real-world contexts.

4. Model with mathematics. Students create models using tape diagrams, double number lines, manipulatives, tables and graphs to represent real-world and mathematical situations involving ratios and proportions. For example, students will examine the relationships between slopes of lines and ratio tables in the context of given situations.

5. Use appropriate tools strategically. Students use visual representations such as the coordinate plane to show the constant of proportionality.

6. Attend to precision. Students attend to the ratio and rate language studied in grade 6 to represent and solve problems involving rates and ratios.

7. Look for and make use of structure. Students look for patterns that exist in ratio tables in order to make connections between the constant of proportionality in a table with the slope of a graph.

8. Look for and express regularity in repeated reasoning. Students formally begin to make connections between covariance, rates, and representations showing the relationships between quantities.

STANDARDS FOR MATHEMATICAL CONTENT

Analyze proportional relationships and use them to solve real-world and mathematical problems.

MGSE7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction (1/2)/(1/4) miles per hour, equivalently 2 miles per hour.

MGSE7.RP.2 Recognize and represent proportional relationships between quantities.

MGSE7.RP.2a Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

MGSE7.RP.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

MGSE7.RP.2c Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.

MGSE7.RP.2d Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1,r)$ where r is the unit rate.

MGSE7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, and fees.

MGSE7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

BIG IDEAS

- Fractions, decimals, and percents can be used interchangeably.
- Ratios and rates use multiplication/division to represent relationships between two quantities.
- The constant of proportionality is also considered to be the unit rate.

ESSENTIAL QUESTIONS

- How can you compute ratios of length in like or different units?
- How can you compute unit rates involving rational numbers, fractions and complex fractions?
- How do I interpret a unit rate (using words and mathematically)?
- What strategies can be used to compare ratios?
- How do I verify if two quantities are directly proportional?
- How can I use tables, graphs or equations to determine whether a relationship is proportional?

- How do I interpret a distance time graph and determine a point of intersection?
- How can models be used to solve percent problems?
- How do I apply mental math strategies to solve percent problems?
- How are distances and measurements translated into a map or scale drawing?
- How do I determine the an appropriate scale for the area (such as my yard or school) that I am measuring and mapping?
- How is the unit rate represented in tables, graphs, equations and diagrams?
- How is unit rate computed in real-world problems?
- How are ratios and their relationships used to solve real world problems?
- How do I solve and interpret solutions of real-world percent problems?
- How do I utilize percent of increase and decrease as an aspect of multiplication?
- How does my understanding of unit rate save me money?
- How can I determine the unit rate for a product that I might purchase?

CONCEPTS AND SKILLS TO MAINTAIN

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- number sense
- computation with whole numbers and decimals, including application of order of operations
- knowledge of equivalent fractions
- addition and subtraction of common fractions with like denominators
- measuring length and finding perimeter and area of rectangles and squares
- characteristics of 2-D and 3-D shapes

FLUENCY

It is expected that students will continue to develop and practice strategies to build their capacity to become fluent in mathematics and mathematics computation. The eventual goal is automaticity with math facts. This automaticity is built within each student through strategy development and practice. The following section is presented in order to develop a common understanding of the ideas and terminology regarding fluency and automaticity in mathematics:

Fluency: Procedural fluency is defined as skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Fluent problem solving does not necessarily mean solving problems within a certain time limit, though there are reasonable limits on how long computation should take. Fluency is based on a deep understanding of quantity and number.

Deep Understanding: Teachers teach more than simply "how to get the answer" and instead support students' ability to access concepts from a number of perspectives. Therefore students are able to see math as more than a set of mnemonics or discrete procedures. Students demonstrate deep conceptual understanding of foundational mathematics concepts by applying them to new situations, as well as writing and speaking about their understanding.

Memorization: The rapid recall of arithmetic facts or mathematical procedures. Memorization is often confused with fluency. Fluency implies a much richer kind of mathematical knowledge and experience.

Number Sense: Students consider the context of a problem, look at the numbers in a problem, make a decision about which strategy would be most efficient in each particular problem. Number sense is not a deep understanding of a single strategy, but rather the ability to think flexibly between a variety of strategies in context.

Fluent students: flexibly use a combination of deep understanding, number sense, and memorization. are fluent in the necessary baseline functions in mathematics so that they are able to spend their thinking and processing time unpacking problems and making meaning from them. are able to articulate their reasoning. find solutions through a number of different paths.

For more about fluency, see: http://www.youcubed.org/wp-content/uploads/2015/03/FluencyWithoutFear-2015.pdf and: http://joboaler.com/timed-tests-and-the-development-of-math-anxiety/

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for middle school students. **Note – Different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.** The definitions below are from the CCSS glossary http://www.corestandards.org/Math/Content/mathematics-glossary/glossary, when applicable.

Visit http://intermath.coe.uga.edu or http://mathworld.wolfram.com to see additional definitions and specific examples of many terms and symbols used in grade 7 mathematics.

- **Constant of Proportionality**
- **Equivalent Fractions**
- **Fraction**

- **Multiplicative inverse**
- **Percent rate of change**
- **Proportion**
- **Ratio**
- **Unit Rate**
- **Scale factor**

FORMATIVE ASSESSMENT LESSONS (FAL)

Formative Assessment Lessons are intended to support teachers in formative assessment. They reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.

More information on Formative Assessment Lessons may be found in the Comprehensive Course Guide.

SPOTLIGHT TASKS

A Spotlight Task has been added to each CCGPS mathematics unit in the Georgia resources for middle and high school. The Spotlight Tasks serve as exemplars for the use of the Standards for Mathematical Practice, appropriate unit-level Common Core Georgia Performance Standards, and research-based pedagogical strategies for instruction and engagement. Each task includes teacher commentary and support for classroom implementation. Some of the Spotlight Tasks are revisions of existing Georgia tasks and some are newly created. Additionally, some of the Spotlight Tasks are 3-Act Tasks based on 3-Act Problems from Dan Meyer and Problem-Based Learning from Robert Kaplinsky.

3-ACT TASKS

A Three-Act Task is a whole group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three.

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

TASKS

Task Name	Task Type *Grouping Strategy*	Content Addressed	Standards Addressed
The Fastest	3-Act Task *Individual or Partner*	Determining the appropriate rate to use when solving a problem	MGSE7.RP.1
What is Unit Rate?	Learning Task *Individual or Small Group*	Introduction and application of writing rates numerically and verbally.	MGSE7.RP.1
Analyzing and Applying Unit Rate	Performance Task *Individual or Partner*	Ability to determine the appropriate rate to use when solving a real life problem	MGSE7.RP.1
Orange Fizz Experiment	Learning Task *Individual or Small Group*	Task requires use of ratios, proportions and proportional reasoning.	MGSE.7.RP.1 MGSE.7.RP.2 MGSE.7.RP.3
Thumbs on Fire	3-Act Task *Individual or Partner*	Solving problems involving proportions	MGSE.7.RP.1 MGSE.7.RP.2 MGSE.7.RP.3
Proportion and Non-Proportion (FAL)	Formative Assessment Lesson *Partner/Small Group*	Comparing proportions and solving using efficient methods	MGSE7.RP.1 MGSE7.RP.2
Nate & Natalie's Walk	Performance Task *Small Group*	Students will determine whether a proportional relationship exists through the use of tables, graphs, or equations.	MGSE.7.RP.2
Buses	Short Cycle Task	Analyzing and creating a visual relationship for distance, rate, and time	MGSE7.RP.2d
Sphero Draw and Drive	3-Act Task *Individual or Partner*	Solving problems involving proportions	MGSE.7.RP.2 MGSE.7.RP.3 MGSE.7.G.1

Comparing Strategies for Proportion Problems (FAL)	Formative Assessment Lesson *Partner/Small Group*	Comparing unit rates and scales in order to compare	MGSE7.RP.1 MGSE7.RP.2 MGSE7.RP.3
Creating a Scale Map	Performance Task *Small Group*	Through mapping an area students will create a scale drawing with an interpretive scale.	MGSE.7.RP.1 MGSE.7.RP.2 MGSE.7.G.1
Drawing to Scale: Designing a Garden	Formative Assessment Lesson	Scale factor Solving problems involving proportions	MGSE.7.RP.1 MGSE.7.RP.2 MGSE.7.G.1
Fish in a Lake	Spotlight Task *Individual or Pairs*	Solving problems involving proportions	MGSE.7.RP.2 MGSE.7.RP.3
Patterns & Percentages	Learning Task *Small Group*	Introduction to percent problems using bar models, ratios & proportional reasoning.	MGSE.7.RP.2 MGSE.7.RP.3
Increasing and Decreasing Quantities (FAL)	Formative Assessment Lesson *Partner/Small Group*	Students will use percent of increase and decrease to compare values	MGSE7.RP.1 MGSE7.RP.2 MGSE7.RP.3
25% Sale	Short Cycle Task	Decreasing percent of a number	MGSE7.RP.1 MGSE7.RP.2 MGSE7.RP.3
Ice Cream	Short Cycle Task	Finding percent of a number to determine profit	MGSE7.RP.1 MGSE7.RP.2 MGSE7.RP.3
Which Is The Better Deal?	Culminating Task *Individual or Pairs*	Unit rates are used to determine the most cost effective products.	MGSE.7.RP.1 MGSE.7.RP.3

The Fastest

Back to task table

Task adapted from www.mikewiernicki.com

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction (1/2)/(1/4) miles per hour, equivalently 2 miles per hour.

MGSE7.RP.2 Recognize and represent proportional relationships between quantities.

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MGSE7.RP.2c Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.

MGSE7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, and fees.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

ESSENTIAL QUESTIONS

How can you compute ratios of length in like or different units?

MATERIALS REQUIRED

- Videos for The Fastest – 3-Act task
- Recording sheet (attached)

TEACHER NOTES

In this task, students will watch the video, then tell what they noticed. They will then be asked to discuss what they wonder or are curious about. These questions will be recorded on a class chart or on the board. Students will then use mathematics to answer their own questions. Students will be given information to solve the problem based on need. When they realize they don't have the information they need, they ask for it, it is given to them.

TASK DESCRIPTION

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

ACT 1:
Watch the video: https://youtu.be/G7LKM0DvSSo

Ask students what they noticed and what they wonder. Record responses.
Suggested questions:
#1 How much faster is the cheetah's average speed over 100 meters than Usain Bolt's average speed over 100 meters?
#2 If the cheetah and Usain Bolt were to race against each other in the 100-meter dash, approximately where would Usain Bolt be when the cheetah crossed the finish line?

Estimate. Write an estimate that is too high and an estimate that is too low.

ACT 2:
The following information is provided for students as they ask for it.

https://youtu.be/0XWxu4OuP-s

ACT 3
Students will compare and share solution strategies.

- Reveal the answer. Discuss the theoretical math versus the practical outcome.
- How appropriate was your initial estimate?
- Share student solution paths. Start with most common strategy.
- Revisit any initial student questions that weren't answered.

DIFFERENTIATION

Extension: Students requiring an extension to this task may wish to compare another animal to the fastest woman - https://www.youtube.com/watch?v=zoqqcXNSgf0 .

Intervention: Students needing support may need additional scaffolding questions so they can determine the proportional relationships needed to answer their questions.

The Fastest Name:______________________

Adapted from Andrew Stadel

ACT 1

What did/do you notice?

What questions come to your mind?

Main Question:____

Estimate the result of the main question? Explain?

Place an estimate that is too high and too low on the number line

Low estimate *Place an "x" where your estimate belongs* High estimate

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimate using this information:______________________________

Act 2 (con’t)
Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.

What is Unit Rate?

Back to task table

Adapted from *Cramer, K.., Behr, M., & Bezuk, N. (1989). Proportional relationships and unit rates. Mathematics Teacher, 82 (7), 537-544.* *http://www.cehd.umn.edu/ci/rationalnumberproject/89_4.html*

Students will build upon their understanding of the unit rates associated with a proportional relationship. Students will also develop the ability to determine the appropriate rate to use in solving a problem and to use the corresponding unit rate to solve missing-value problems.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction (1/2)/(1/4) miles per hour, equivalently 2 miles per hour.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning

COMMON MISCONCEPTIONS

A common error in setting up proportions is placing numbers in incorrect locations. This is especially easy to do when the order in which quantities are stated in the problem is switched within the problem statement.

ESSENTIAL QUESTIONS:

- How can you compute unit rates involving rational numbers, fractions and complex fractions?

MATERIALS:

- Student recording sheet

GROUPING

Individual / Partner

TASK DESCRIPTION:

Begin the lesson with a Number Talk. Number talks are a great way for students to use mental math to solve and explain a variety of math problems. A Number Talk is a short, ongoing daily routine that provides students with meaningful ongoing practice with computation. Number Talks should be structured as short sessions alongside (but not necessarily directly related to) the ongoing math curriculum. It is important to keep Number Talks short, as they **are not intended to replace current curriculum or take up the majority of the time spent on mathematics**.

In fact, teachers need to spend only 5 to 15 minutes on Number Talks. Number Talks are most effective when done every day. As previously stated, the primary goal of Number Talks is computational fluency. Students develop computational fluency while thinking and reasoning like mathematicians. When they share their strategies with others, they learn to clarify and express their thinking, thereby developing mathematical language. This in turn serves them well when they are asked to express their mathematical processes in writing. In order for students to become computationally fluent, they need to know particular mathematical concepts that go beyond what is required to memorize basic facts or procedures.

All Number Talks follow a basic six-step format. The format is always the same, but the problems and models used will differ for each number talk.

1. **Teacher presents the problem.** Problems are presented in a word problem or a written algorithm.

2. **Students figure out the answer.** Students are given time to figure out the answer. To make sure students have the time they need, the teacher asks them to give a "thumbs-up" when they have determined their answer. The thumbs up signal, given at chest level, is unobtrusive- a message to the teacher, not the other students.

3. **Students share their answers**. Four or five students volunteer to share their answers and the teacher records them on the board.

4. **Students share their thinking.** Three or four students volunteer to share how they got their answers. (Occasionally, students are asked to share with the person(s) sitting next to them.) The teacher records the student's thinking.

5. **The class agrees on the "real" answer for the problem.** The answer that together the class determines is the right answer is presented as one would the results of an experiment. The answer a student comes up with initially is considered a conjecture. Models and/or the logic of the explanation may help a student see where their thinking went wrong, may help them identify a step they left out, or clarify a point of confusion. There should be a sense of confirmation or clarity rather than a feeling that each problem is a test to see who is right and who is wrong. A student who is still unconvinced of an answer should be encouraged to keep thinking and to keep trying to understand. For some students, it may take one more experience for them to understand what is happening with the numbers and for others it may be out of reach for some time. The mantra should be, "If you are not sure or it doesn't make sense yet, keep thinking."

6. **The steps are repeated for additional problems**.

   Similar to other procedures in your classroom, there are several elements that must be in place to ensure students get the most from their Number Talk experiences. These elements are:

1. A safe environment
2. Problems of various levels of difficulty that can be solved in a variety of way
3. Concrete models
4. Opportunities to think first and then check

5. Interaction
6. Self-correction

For this Number Talk, begin with the following problem, “6 melons cost $12. How much does one melon cost?” Record the problem on the far left side of the board. Provide students with wait time as they work to mentally solve this problem. When majority of the students have given the “thumbs up” signal, call on several students (3-4) to share their answer and the strategy they used to solve. Record the information provided by the students exactly how it is told to you. It is important to allow students ownership of their thinking.

Record, “3 watermelons cost $15. How much does one watermelon cost?” on the board next to the previous problem. Provide students with wait time as they work to mentally solve this problem. When majority of the students have given the “thumbs up” signal, call on several students (3-4) to share their answer and the strategy they used to solve. Record the information provided by the students exactly how it is told to you.

Record, “3 apples cost $.99. How much does 1 apple cost?” on the board towards the far right. Provide students with wait time as they work to mentally solve this problem. When majority of the students have given the “thumbs up” signal, call on several students (3-4) to share their answer and the strategy they used to solve. Record the information provided by the students exactly how it is told to you.

At the end of the Number Talk, discuss the strategies used to find the answers. Some of the strategies students may use are relationship between multiplication and division, division, guess and check and skip counting. Talk with the students about which strategy was most efficient (quick, easy and accurate).

Allow for a maximum of 15 minutes to conduct the Number Talk before moving into the lesson.

Explain to students, in the Number Talks, strategies were used to identify a **unit rate**.
Revisit the first problem from the Number Talk. Have students identify the rate (6/12), then identify the unit rate (1/2). Have students define a unit rate.

On the recording sheet, students analyze a real life situation to create two versions of a unit rate. Then, students need to analyze the two possible rates and determine which is the most appropriate to the given problem and use this rate to find possible cost for other values within the given problem.

Selecting the Appropriate Unit Rate

At Ralph's fruit stand ½ pound of grapes cost 90 cents. You want to buy 5 pounds of grapes. How much will they cost?

1. What are the two possible rates for this problem?

   ***Solution:** ½ pound/90 cents   90 cents/ ½ pound*

2. Show each rate as a unit rate.
   ***Solution:** 0.006 pounds of per 1 cent   180 cents per 1 pound of grapes*

3. What does each unit rate tell you?
   ***Solution:** The pounds of grapes for 1 cent*
   The number of cents for one pound of grapes

4. Which unit rate will help you solve the problem?
   ***Solution:** 180 per 1 pound of grapes*

5. If it costs 180 cents to buy 1 pound of grapes, how much will 2 pounds cost? 4 pounds? Complete the table below.

POUNDS OF GRAPES	COST IN CENTS
1	*180*
2	*360*
3	*540*

What pattern do you see? ***The cost in cents increases by 180 cents each time.***

Multiply the pounds of grapes by 180 cents to get the total cost

6. Since you know the unit price, write a number sentence for the cost of five pounds of grapes. Write an equation for the cost of any pounds of grapes.
   (180 cents per pound)(5 pounds) = $9.00
   (Unit rate)(pounds of grapes) = total cost
   180x = y where x is the cost per pound of grapes and y is the number of pounds.

What Is The Unit Rate?

Selecting the Appropriate Unit Rate

Based on your understanding of the models during the Number Talk, how would you explain or define a unit rate?

At Ralph's fruit stand ½ pound of grapes cost 90 cents. You want to buy 5 pounds of grapes. How much will they cost?

1. What are the two possible rates for this problem?

2. Show each rate as a unit rate.

3. What does each unit rate tell you?

4. Which unit rate will help you solve the problem?

5. If it costs 180 cents to buy 1 pound of grapes, how much will 2 pounds cost? 4 pounds? Complete the table below. Then, describe the pattern you see in the chart.

POUNDS OF GRAPES	COST IN CENTS
1	180
2	
3	

6. Since you know the unit price, write a number sentence for the cost of five pounds of grapes. Write an equation for the cost of any pounds of grapes.

Analyzing and Applying Unit Rate

Back to task table

Students will build upon their understanding of the unit rates associated with a proportional relationship. Students will also develop the ability to determine the appropriate rate to use in solving a problem and to use the corresponding unit rate to solve missing-value problems.
The approximate time for this task is 1-2 class periods.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction (1/2)/(1/4) miles per hour, equivalently 2 miles per hour.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving the.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.

BACKGROUND KNOWLEDGE

- In the previous lesson students were introduced to the idea of unit rates.
- In 6^{th} grade, students learning to solve unit rate problems including those involving unit pricing and constant speed.

COMMON MISCONCEPTIONS

A common error in setting up proportions is placing numbers in incorrect locations. This is especially easy to do when the order in which quantities are stated in the problem is switched within the problem statement.

ESSENTIAL QUESTIONS

- How do I interpret a unit rate (using words and mathematically)?

MATERIALS

- Student recording sheets

GROUPING

Individual/Partner

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

PART 1

Comments

Students need the opportunity to work with manipulatives on their own or with a partner in order to develop the strategies for finding and interpreting unit rates. From the manipulatives, students will be able to move to pictorial representations of the display, then more abstract representations (such as sketches), and finally to abstract representation of rate equivalents. It is important to remember that this progression begins with concrete representations using manipulatives.

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

Task Directions

Act I – Whole Group - Pose the conflict and introduce students to the scenario by showing Act I picture.

1. Students are shown the Sales Ad image.

2. Ask students what they wonder about and what questions they have about what they saw. Students should share with each other first, and then the teacher records these questions (think-pair-share). The teacher may need to guide students so that the questions generated are math-related.
Anticipated question:
Which option of Crest Pro Rinse is the better buy?

3. Ask students to estimate answers to the question (think-pair-share). Students will write their best estimate, along with an explanation of why they chose their estimate.

When students record their estimates on a number line, it will provide an informal assessment of students understanding of quantities and their value.

Act II – **Student Exploration** - Provide additional information as students work toward solutions to their questions.

1. Ask students to determine what additional information they will need to solve their questions. The teacher provides that information only when students ask for it.:

This information is apparent within the image, however, the image will not remain posted and students may need this information to help them answer the question(s) from Act I. Only provide the information to students who inquire about it.

2. Ask students to work to answer the questions they created in Act I. The teacher provides guidance as needed during this phase by asking questions such as:
    - Can you explain what you've done so far?
    - What strategies are you using?
    - What assumptions are you making?
    - What tools or models may help you?
    - Why is that true?
    - Does that make sense?

Doing this time, students will apply their understanding of unit rate to identify unit rates of particular products at the grocery store. Students may also begin to look at how to change a unit rate to a rate through multiplication. Look for the strategies students use for each to share in Act III.

For students who struggle during this act, have them refer to the strategies used in the Number Talk for the previous lesson. These strategies may be the relationship between multiplication and division, finding an equivalent expression, division or the less efficient skip counting.

Act III – Whole Group - Share student solutions and strategies as well as Act III solution.

1. Ask students to present their solutions and strategies.
2. Share solution in Act III solution.

PART 2
Applying the Unit Rate Approach

In each problem, record the rate appropriate for the question asked, find the corresponding unit rate, write a short sentence interpreting the unit rate, and use this rate to find the solution to the problem.

1. Anne is painting her house light blue. To make the color she wants, she must add 3 cans of white paint to every 2 cans of blue paint. How many cans of white paint will she need to mix with 6 cans of blue?

   Rate needed (white/ blue) ____***3 cans white / 2 cans blue***__

   Unit Rate ___*1.5 cans white / 1 can blue*____

   Interpretation of unit rate ___**Anne should mix 1.5 cans of white paint with each can of blue.**__

   Solution: ***(unit rate) (number of items) = total***
   (1.5)(6 blue) = 9 white cans of paint

2. Ryan is making a fruit drink. The directions say to mix 3 cups of water with 1 ½ scoops of powdered fruit mix. How many cups of water should he use with 6 scoops of fruit mix?

   Rate needed ___***3 cups water / 1 ½ scoops mix***___________

   Unit Rate _____***2 cups water / 1 scoop mix***______________

   Interpretation of Unit Rate __***Ryan should use 2 cups of water for each scoop of mix.***________

   Solution: **(*unit rate) (number of scoops of mix) = number of cups of water***
   (2)(6) = 12 c. of water

3. Donna is running around a track. It takes her 3 ½ minutes to run ¼ of a lap. If she keeps running at the same speed, how long will it take her to run 5 laps?

   Rate needed ______***3 ½ minutes / ¼ lap***___________

   Unit rate ____***14 minutes / 1 lap***________

   Interpretation of unit rate _____***Donna runs 1 lap in 14 minutes***___________

   Solution: ***(14 min. /lap) (5 laps) = 70 minutes***

4. Mark's model train can go 1/3 laps around its track in 2 minutes. If it runs at the same speed, how many laps can the train go in 9 minutes?

   Rate needed _____***1/2 lap / 2 minutes***_______

   Unit Rate ____***¼ laps per minute***________________

   Interpretation of Unit Rate ___***Mark's train travels ¼ laps each minute***____________

   Solution: ***(¼ laps / minute) (9 minutes) = 2 ¼ laps***

Analyzing And Applying Unit Rate

Name: ________________________
Adapted from Andrew Stadel

PART 1

ACT 1

What questions come to your mind?

Main Question: ____

What is your 1st estimate and why? On an empty number line, record an estimate that is too low and an estimate that is too high.

ACT 2

What information would you like to know or need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimation with this information: ______________________________

Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

APPLYING THE UNIT RATE APPROACH

<u>PART 2</u>

In each problem, record the rate appropriate for the question asked, find the corresponding unit rate, write a short sentence interpreting the unit rate, and use this rate to find the solution to the problem.

1. Anne is painting her house light blue. To make the color she wants, she must add 3 cans of white paint to every 2 cans of blue paint. How many cans of white paint will she need to mix with 6 cans of blue?

   Rate needed (white/ blue) ______________________ Unit Rate ____________________

   Interpretation of unit rate ____

   Solution:

2. Ryan is making a fruit drink. The directions say to mix 3 cups of water with 1 ½ scoops of powdered fruit mix. How many cups of water should he use with 6 scoops of fruit mix?

   Rate needed _______________________________ Unit Rate _______________

   Interpretation of Unit Rate ____

   Solution:

3. Donna is running around a track. It takes her 3 ½ minutes to run ¼ of a lap. If she keeps running at the same speed, how long will it take her to run 5 laps?

   Rate needed _______________________________ Unit rate__________________________

   Interpretation of unit rate ____

   Solution:

4. Mark's model train can go $1/3$ laps around its track in 2 minutes. If it runs at the same speed, how many laps can the train go in 9 minutes?

   Rate needed _______________________________ Unit Rate________________________

   Interpretation of Unit Rate ____

   Solution:

** Orange Fizz Experiment

Back to task table

In the task, students analyze and solve an open-ended problem where they are asked to compare the strength of the orange taste of three drink mixes.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction (1/2)/(1/4) miles per hour, equivalently 2 miles per hour.

MGSE7.RP.2 Recognize and represent proportional relationships between quantities.

MGSE7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, and fees.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.

COMMON MISCONCEPTIONS

A common error is to reverse the position of the variables when writing equations. Students may find it useful to use variables specifically related to the quantities rather than using x and y.

Constructing verbal models can also be helpful. They can use this verbal model to construct the equation. Students can check their equation by substituting values and comparing their results to the table. The checking process helps student revise and recheck their model as necessary.

ESSENTIAL QUESTIONS:

- What strategies can be used to compare ratios?

MATERIALS:

- Student Task sheet

GROUPING:

Individual / Grouping

TASK COMMENTS

Approaches to the task may include a range of different strategies. Students will investigate how ratios can be formed and scaled up to find equivalent ratios. In addition, students will use proportional reasoning to decide how to use the different mixes to make drinks for 200 people.

The task is a challenging problem for students and at least 1 to 2 class periods should be allowed for students to explore and have a whole-class discussion on the task.

TASK DESCRIPTION

Begin the lesson with a Number Talk. Number talks are a great way for students to use mental math to solve and explain a variety of math problems. A Number Talk is a short, ongoing daily routine that provides students with meaningful ongoing practice with computation. Number Talks should be structured as short sessions alongside (but not necessarily directly related to) the ongoing math curriculum. It is important to keep Number Talks short, as they **are not intended to replace current curriculum or take up the majority of the time spent on mathematics**.

In fact, teachers need to spend only 5 to 15 minutes on Number Talks. Number Talks are most effective when done every day. As previously stated, the primary goal of Number Talks is computational fluency. Students develop computational fluency while thinking and reasoning like mathematicians. When they share their strategies with others, they learn to clarify and express their thinking, thereby developing mathematical language. This in turn serves them well when they are asked to express their mathematical processes in writing. In order for students to become computationally fluent, they need to know particular mathematical concepts that go beyond what is required to memorize basic facts or procedures.

All Number Talks follow a basic six-step format. The format is always the same, but the problems and models used will differ for each number talk.

1. **Teacher presents the problem.** Problems are presented in a word problem or a written algorithm.

2. **Students figure out the answer.** Students are given time to figure out the answer. To make sure students have the time they need, the teacher asks them to give a “thumbs-up” when they have determined their answer. The thumbs up signal, given at chest level, is unobtrusive- a message to the teacher, not the other students.

3. **Students share their answers**. Four or five students volunteer to share their answers and the teacher records them on the board.

4. **Students share their thinking.** Three or four students volunteer to share how they got their answers. (Occasionally, students are asked to share with the person(s) sitting next to them.) The teacher records the student's thinking.

5. **The class agrees on the "real" answer for the problem.** The answer that together the class determines is the right answer is presented as one would the results of an experiment. The answer a student comes up with initially is considered a conjecture. Models and/or the logic of the explanation may help a student see where their thinking went wrong, may help them identify a step they left out, or clarify a point of confusion. There should be a sense of confirmation or clarity rather than a feeling that each problem is a test to see who is right and who is wrong. A student who is still unconvinced of an answer should be encouraged to keep thinking and to keep trying to understand. For some students, it may take one more experience for them to understand what is happening with the numbers and for others it may be out of reach for some time. The mantra should be, "If you are not sure or it doesn't make sense yet, keep thinking."

6. **The steps are repeated for additional problems**.

Similar to other procedures in your classroom, there are several elements that must be in place to ensure students get the most from their Number Talk experiences. These elements are:

1. A safe environment
2. Problems of various levels of difficulty that can be solved in a variety of way
3. Concrete models
4. Opportunities to think first and then check
5. Interaction
6. Self-correction

For this Number Talk, begin with the following problem, "There are 12 boys and 18 girls in Ms. Dade's class. What is the boy to girl ratio in the class?" Record the problem on the far left side of the board. Provide students with wait time as they work to mentally solve this problem. When majority of the students have given the "thumbs up" signal, call on several students (1-2) to share their answer and the strategy they used to solve. Record the information provided by the students exactly how it is told to you. It is important to allow students ownership of their thinking.

Record, "Mr. Hill has 30 students. 14 of the students are boys. What is the ratio of boys in the class?" on the board next to the previous problem. Provide students with wait time as they work to mentally solve this problem. When majority of the students have given the "thumbs up" signal, call on several students (1-2) to share their answer and the strategy they used to solve. Record the information provided by the students exactly how it is told to you.

Record, "The ratio for male and female students in Ms. Dade's class is 12:18. The ratio of males in Mr. Hill's entire class is 14 to 30. Whose class is composed of more boys?" on the board towards the right. Provide students with wait time as they work to mentally solve this problem. When majority of the students have given the "thumbs up" signal, call on several students (3-4) to share their answer and the strategy they used to solve. Record the information provided by the students exactly how it is told to you.

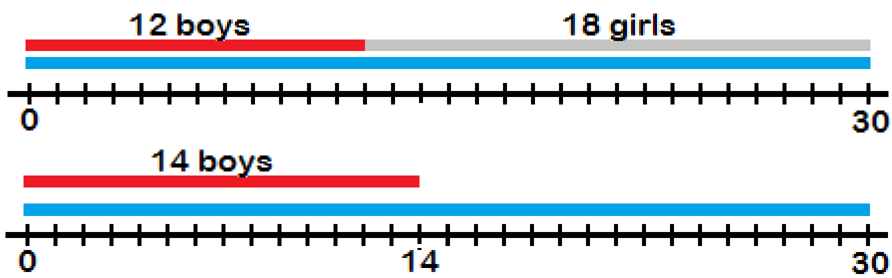
Record, "Mrs. Ford's class has 14 boys and 16 girls. Of the 3 classes, whose class is composed of the most boys?" on the board towards the far right. Provide students with wait time as they work to mentally solve this problem. When majority of the students have given the "thumbs up" signal, call on several students (3-4) to share their answer and the strategy they used to solve. Record the information provided by the students exactly how it is told to you.

At the end of the Number Talk, discuss the strategies used to find the answers. Some of the strategies students may use are relationship between multiplication and division, division, guess and check and skip counting. Talk with the students about which strategy was most efficient (quick, easy and accurate).
Prior to the task students need to discuss and explore making comparisons with ratios, percents, and fractions. Models and drawings, as illustrated below may facilitate student understanding.

Relate the strategies from the introductory problems to the task where students will be comparing the mixes. Make these connections during the whole-group discussion. Students should be able to see how each form, ratios, percents, and fraction, provides information needed to derive one of the other forms.

1. Compare the ratio you found for Ms. Dade's class to the ratio you found for Mr. Hill's class. What is alike or different about each?

#1 is comparing part-to-part and #2 is comparing part-to-whole. Discuss how to compare the two classes using tables, percents, common denominators or models. When #1 is set up as part-to-whole (12: 30), a comparison can be made easily using common denominators. The model below also illustrates the comparison.



2. Compare the ratio you found for the number pets in your house to the ratio you found for the number of pets in Darla's house. What is alike or different about each?

Comparison:

#1 is comparing part-to-part and #2 is comparing part-to-whole. Discuss how to compare the two classes using tables, percents, common denominators or models. The model below compares the ratios using a bar model and percents.

Orange Fizz Experiment

A famous cola company is trying to decide how to change their drink formulas to produce the best tasting soda drinks on the market. The company has three different types of formulas to test with the public. The formula consists of two ingredients: orange juice concentrate and carbonated water.

You are a scientist working for this company, and you will get paid a large commission if you can find the right formula that will sell the best. Your job is to find out which of the formulas is the best tasting of the flavors.
Using the company's new formulas, you must follow the recipe to the strict guidelines:

Formula A: 1 tablespoons of orange concentrate to 2 tablespoons of carbonated water

Formula B: 2 tablespoons of orange concentrate to 5 tablespoons of carbonated water

Formula C: 2 tablespoons of orange concentrate to 3 tablespoons of carbonated water

Part A: Using part-to-whole comparison ***Suggestion Only***

***Solution Comments:** It is important to emphasize the difference between part-to-part and part-to whole comparison. Teachers may want to facilitate discussion centered on which strategy works best for Part A and which works for Part B.*

1. Which formula will make a drink that has the *strongest* orange taste? Show your work and explain your choice.
   Solution:
   The approaches to the solution may vary. Students should be able to show and explain that Formula C will have the strongest orange taste since the ratio of concentrate to orange fizz (part-to-whole) will be 2:5. Formulas A and B have ratios of concentrate to juice of 1:3 and 2:7 respectively. Students may justify their findings by comparing fractions through like denominators, using models, and/or percents or decimals.
   For example:
   They could compare two ratios at a time: (Formula A) $\frac{1}{3} = \frac{7}{21}$

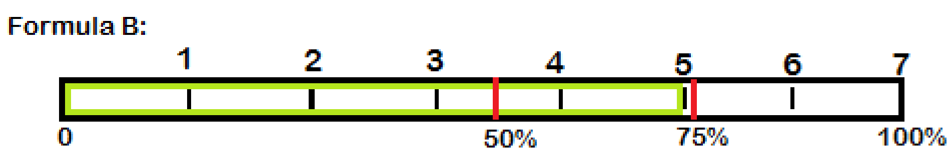
   (Formula B) $\frac{2}{7} = \frac{6}{21}$

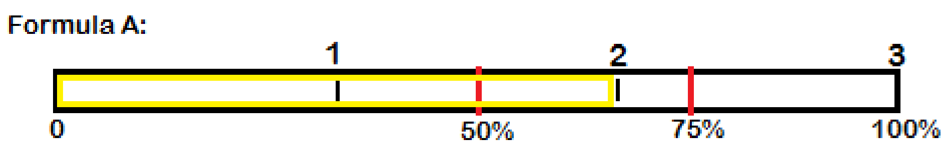
   Then compare Formula A to Formula C, since A was larger than B.
   (Formula A) $\frac{1}{3} = \frac{5}{15}$

   (Formula C) $\frac{2}{5} = \frac{6}{15}$

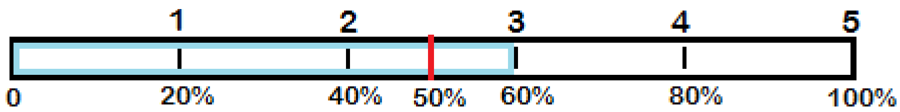
   Therefore, the drink with the highest ratio of concentrate to Orange Fizz would have the strongest orange taste.

2. Which formula has the highest percentage of carbonated water in the mixture? Estimations may be used. Show your work and justify your answer.
   Solution:
   Students could use bar models to compare through estimation the highest percentage of carbonated water in the mixture. Solution approaches may vary. Students may realize that they can use the information from #1 to help them solve the problem. For example, since Formula B had the lowest concentrate then it must have the highest amount of carbonated water among the three formulas. Formula B has 5 to 7 ratio of carbonated water to juice. Students may estimate percentages using a bar model or students may calculate decimals or precise percentages using proportions.





Formula C:



Part B: Using part-to-part comparison ***Suggestion ONLY!***

1. For researchers to test their product, they will need to produce enough of each of the three drink formulas to take to various locations around the area for taste testing. Researchers would like for *at least 100 people* to sample each formula.
   Each sample will contain 1 of a cup of liquid.

   Formula A: 1 cup of concentrate to 2 cups of carbonated water

   Formula B: 2 cups of concentrate to 5 cups of carbonated water

   Formula C: 2 cups of concentrate to 3 cups of carbonated water

   Fill in the table to determine the least amount of concentrate and carbonated water that you would have to use to serve 1 cup servings to 100 people.
   All of the following tables are used to help students understand how the relationship between equivalent ratios may be beneficial within a real-world context.

Formula A:		
Orange Concentrate (cups)	Carbonated Water (cups)	Total Amount (cups/servings) ***Sum of Column 1 and 2***
1 ***Given***	2 ***Given***	3 cups (3 servings)
2 ***Multiply O.C. by 2***	4 ***Multiply C.W. by 2***	6 cups (6 servings)
3 ***Multiply O.C. by 3***	6 ***Multiply C.W. by 3***	9 cups (9 servings)
…	…	,,,
17 ***Multiply O.C. by 17***	34 ***Multiply C.W. by 17***	51 cups (51 servings)
…	…	…
33 ***Multiply O.C. by 33***	66 ***Multiply C.W. by 33***	***99*** cups (***99*** servings)
34 ***Multiply O.C. by 34***	***68***	***102*** cups (***102*** servings)
35	70 ***Multiply C.W. by 35***	***105*** cups (***105*** servings)

Solution
The numbers needed to complete the table are found by creating equivalent ratios. Students are asked to determine which amount will provide enough servings for at least 100 people.

Students may continue to add 1:2:3 to each row until they reach 17:34:51, or they may notice that they can multiply a row by a scale factor to get to their result more quickly.

I. How much orange concentrate and carbonated water is needed to serve at least 100 people?

a. Orange Concentrate
At least 34 cups

b. Carbonated Water
At least 68 cups

Formula B:		
Concentrate (cups)	Carbonated Water (cups)	Total Amount (cups/servings)
2	5	7 cups (7 servings)
4	10	14 cups (14 servings)
8	***20***	28 cups (28 servings)
…	…	,,,
2(7)=14	35	***49*** cups (49 servings)
…	…	…
26	***5(13)=65***	***91*** cups (***91*** servings)
2(14)=28	***5(14)=70***	98 cups (98 servings)
2(15)=30	***5(15)=75***	***105*** cups (***105*** servings)

II. How much orange concentrate and carbonated water is needed to serve at least 100 people?

c. Orange Concentrate
30 cups (at least)

d. Carbonated Water
75 (at least)

Formula C:		
Orange Concentrate (cups)	Carbonated Water (cups)	Total Amount (cups/servings)
2	3	5 cups (5 servings)
4	6	10 cups (10 servings)
6	***3(3)=9***	***15 cups (15 servings)***
…	…	,,,
20	***30***	***50*** cups (***50*** servings)
…	…	…
36	***54***	***90*** cups (***90*** servings)
38	57	***95*** cups (***95*** servings)
40	***60***	***100*** cups (***100*** servings)

III. How much orange concentrate and carbonated water is needed to serve at least 100 people?

e. Orange Concentrate
40 cups

f. Carbonated Water ***60 cups***

2. Your lab technicians will be bringing you all of the supplies that you will need in order to make the formulas at the sites. In the table below, write the total amount of concentrate and carbonated water.

	Orange Concentrate	**Carbonated Water**
Formula A	***34 cups***	***68 cups***
Formula B	***30 cups***	***75 cups***
Formula C	***40 cups***	***60 cups***
Total (cups)	***104 cups***	***208 cups***

3. Your lab technician calls you that he only has gallon jugs. (Hint: one gallon=16 cups)
   a. How many gallons of orange concentrate do you need to make Formula A? Justify your answer.

This solution requires students to be able to convert between measurements, which ties back to sixth grade content material. Students will need to know that there are 16 cups in 1 gallon. Using this information, students can divide the amount of orange concentrate by the cups per gallon unit to get the total amount of gallons that the lab technicians need to bring.

$$\frac{36\ cups}{16\ cups} = 2\frac{1}{4}\ gallons\ of\ orange\ concentrate$$

   b. How many gallons of carbonated water do you need to make Formula B? Justify your answer.

The solution for this part mimics the solution above.

This solution requires students to be able to convert between measurements, which ties back to sixth grade content material. Students will need to know that there are 16 cups in 1 gallon. Using this information, students can divide the amount of orange concentrate by the cups per gallon unit to get the total amount of gallons that the lab technicians need to bring.

$$\frac{70\ cups}{16\ cups} = 4\frac{3}{8}\ gallons\ of\ orange\ concentrate$$

Orange Fizz Experiment

A famous cola company is trying to decide how to change their drink formulas to produce the best tasting soda drinks on the market. The company has three different types of formulas to test with the public. The formula consists of two ingredients: orange juice concentrate and carbonated water.

You are a scientist working for this company, and you will get paid a large commission if you can find the right formula that will sell the best. Your job is to find out which of the formulas is the best tasting of the flavors.

Using the company's new formulas, you must follow the recipe to the strict guidelines:

Formula A: 1 tablespoons of orange concentrate to 2 tablespoons of carbonated water

Formula B: 2 tablespoons of orange concentrate to 5 tablespoons of carbonated water

Formula C: 2 tablespoons of orange concentrate to 3 tablespoons of carbonated water

Part A: *Using part-to-whole comparison*

1. Which formula will make a drink that has the *strongest* orange taste? Show your work and explain your choice.

2. Which formula has the highest percentage of carbonated water in the mixture? Estimations may be used. Show your work and justify your answer.

Part B: *Using part-to-part comparison*

1. For researchers to test their product, they will need to produce enough of each of the three drink formulas to take to various locations around the area for taste testing. Researchers would like for *at least 100 people* to sample each formula.
   Each sample will contain 1 of a cup of liquid.

   Formula A: 1 cup of concentrate to 2 cups of carbonated water

   Formula B: 2 cups of concentrate to 5 cups of carbonated water

   Formula C: 2 cups of concentrate to 3 cups of carbonated water

   Fill in the table to determine the least amount of concentrate and carbonated water that you would have to use to serve 1 cup servings to 100 people.

Formula A:		
Orange Concentrate (cups)	Carbonated Water (cups)	Total Amount (cups/servings)
1	2	3 cups (3 servings)
2	4	6 cups (6 servings)
3	6	9 cups (9 servings)
…	…	,,,
17	34	51 cups (51 servings)
…	…	…
33	66	____ cups (____ servings)
34		____ cups (____ servings)
	70	____ cups (____ servings)

I. How much orange concentrate and carbonated water is needed to serve at least 100 people?

a. Orange Concentrate-
b. Carbonated Water-

Formula B:		
Concentrate (cups)	Carbonated Water (cups)	Total Amount (cups/servings)
2	5	7 cups (7 servings)
4	10	14 cups (14 servings)
8		28 cups (28 servings)
…	…	,,,
	35	_____cups (49 servings)
…	…	…
26		_____ cups (____ servings)
		98 cups (98 servings)
		_____cups (servings)

II. How much orange concentrate and carbonated water is needed to serve at least 100 people?

c. Orange Concentrate-
d. Carbonated Water

Formula C:		
Orange Concentrate (cups)	Carbonated Water (cups)	Total Amount (cups/servings)
2	3	5 cups (5 servings)
4	6	10 cups (10 servings)
…	…	,,,
20		____ cups (____ servings)
…	…	…
36		____ cups (____ servings)
	57	____ cups (____ servings)
		____ cups (____ servings)

III. How much orange concentrate and carbonated water is needed to serve at least 100 people?

e. Orange Concentrate
f. Carbonated Water

2. Your lab technicians will be bringing you all of the supplies that you will need in order to make the formulas at the sites

	Orange Concentrate	**Carbonated Water**
Formula A		
Formula B		
Formula C		
Total (cups)		

3. Your lab technician calls you that he only has gallon jugs. (Hint: one gallon=16 cups)
   a. How many gallons of orange concentrate do you need to make Formula A? Justify your answer.

   b. How many gallons of carbonated water do you need to make Formula B? Justify your answer.

Thumbs on Fire

Back to task table

CONTENT STANDARDS

MGSE7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction (1/2)/(1/4) miles per hour, equivalently 2 miles per hour.

MGSE7.RP.2 Recognize and represent proportional relationships between quantities.

MGSE7.RP.2c Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.

MGSE7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, and fees.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

ESSENTIAL QUESTIONS

- How are proportional relationships used to solve multistep ratio and percent problems?
- How do equations represent proportional relationships?

MATERIALS REQUIRED

3-Act Task taken from: http://gfletchy3act.wordpress.com/thumbs-on-fire/
Act-1 Video link: http://vimeo.com/99371274
Act-2 Video link: http://vimeo.com/99369909
Act-3 Video link: http://vimeo.com/99367439
3-Act Task Recording Sheet

Teacher Notes

This task is designed around a specific context (the world record for texting) while allowing teachers the flexibility of developing the procedures. In the task description below, one way of implementing this task is described although there are many different ways in which this context could be used in a Math 7 class.

In this task, students will watch the video, then tell what they noticed. They will then be asked to discuss what they wonder or are curious about. These questions will be recorded on the board and on student recording sheet. Students will then use mathematics to answer their own questions. Students will be given information to solve the problem based on need. When they realize they don't have the information they need, and ask for it and it will be given to them.

Task Description

<u>Thumbs on Fire (Part 1)</u>
The following 3-Act Task can be found at: http://gfletchy3act.wordpress.com/thumbs-on-fire/

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

ACT 1:
Watch the video: http://vimeo.com/99371274

Students are asked what they noticed in the video. Students record what the noticed or wondered on the recording sheet. Students are asked to discuss and share what they wondered (or are curious about) as related to what they saw in the video.

<u>Important Note:</u> Although the MAIN QUESTION of this lesson is ***How many seconds will it take to complete the following text message?*** It is important for the teacher to not ignore student generated questions and try to answer as many of them as possible (before, during, and after the lesson as a follow up).

Main Question: ***How many seconds will it take to complete the following text message?***
Write down an estimate you know if too high? Too low?

ACT 2:
Students will realize that they do not have enough information to complete the problem. Release the following information to students ONLY AFTER they have identified what information they need.

Watch the video: After watch the following video http://vimeo.com/99369909 students should recognized the following information:
Release this information first:

- **World record time:** 20.53 seconds
- **Time passed in video:** 5.01 seconds
- **Percent complete:** 26.75
- **Number of letters completed:** 42 letters

Release this information as a scaffolding piece and intervention:

- **The passage:** *The razor-toothed piranhas of genera Serrasalmus and Pygocentrus are the most ferocious freshwater fish in the world. In reality they seldom attack a human.*
- **Number of characters in the passage:** 157 characters (spaces included as a character)

Students may work individually for 5-10 minutes to come up with a plan, test ideas, and problem-solve. After individual time, students may work in groups (3-4 students) to collaborate and solve this task together.

ACT 3

Show students the Act-3 Video http://vimeo.com/99367439

- Students will compare and share solution strategies.
- Reveal the answer. Discuss the theoretical math versus the practical outcome.
- How appropriate was your initial estimate?
- Share student solution paths. Start with most common strategy.
- Revisit any initial student questions that weren't answered.

ACT 4

Assuming the individual texting in the video could maintain a constant rate of letters per second, how long would it take to text a 1,500 word essay? Identify an equation that would work for any word essay?

Teacher note: students will need to determine the number of letters the average word has. The decision of how to determine this should be left up to the students.

Task Title:________________________ Name:________________________

Adapted from Andrew Stadel

ACT 1

What did/do you notice?

What questions come to your mind?

Main Question:____

Estimate the result of the main question? Explain?
Place an estimate that is too high and too low on the number line
Low estimate *Place an "x" where your estimate belongs* High estimate

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimate using this information:_______________________________

Act 2 (con't)
Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.

Classifying Proportion and Non-Proportion Situations (FAL)

(Concept Development) **Back to task table**

This lesson unit is intended to help you assess whether students are able to identify when two quantities vary in direct proportion to each other, distinguish between direct proportion and other functional relationships, and solve proportionality problems using efficient methods.

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/lessons.php?unit=7215&collection=8

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction (1/2)/(1/4) miles per hour, equivalently 2 miles per hour.

MGSE7.RP.2 Recognize and represent proportional relationships between quantities.

MGSE7.RP.2a Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

MGSE7.RP.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

MGSE7.RP.2c Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.

MGSE7.RP.2d Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1,r)$ where r is the unit rate.

MGSE7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, and fees.

STANDARDS FOR MATHEMATICAL PRACTICE

This lesson uses all of the practices with emphasis on:
1. Make sense of problems and persevere in solving them.
6. Attend to precision.
8. Look for and express regularity in repeated reasoning.

ESSENTIAL QUESTIONS

- How do I verify if two quantities are directly proportional?

TASK COMMENTS

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Classifying Proportion and Non-Proportion Situations*, is a Formative Assessment Lesson (FAL) that can be found at the website:
http://map.mathshell.org/lessons.php?unit=7215&collection=8

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=1356

Nate & Natalie's Walk

Back to task table

Students will use proportional reasoning to compare the distance that a brother and sister walk.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.RP.2 Recognize and represent proportional relationships between quantities.

MGSE7.RP.2a Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

MGSE7.RP.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

MGSE7.RP.2c Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
4. Model with mathematics.
6. Attend to precision.
8. Look for and express regularity in repeated reasoning.

COMMON MISCONCEPTIONS

- Students possibly get confused with which axis is the x and which one is the y.
- They also don't remember which to plot first x or y.
- When they set up the proportions they get confused as to what numbers go where and what to do with them once they have them.
- Labeling of the axes

ESSENTIAL QUESTIONS

- How can I use tables, graphs or equations to determine whether a relationship is proportional?

MATERIALS

- Nate and Natalie's Walk Task sheet
- Cuisenaire rods

TASK COMMENTS

Prior to doing this performance task, students should understand that graphing is a way to visually represent ratios and proportional relationships. This visual is a tool that can be used to determine the reasonableness of an equation and to draw conclusions about proportional relationships.

TASK DESCRIPTION

Hand out the task and allow students to work individually for 3-5 minutes without intervening. Circulate around the classroom to get an idea about what strategies are being used to solve the problem.
After 3-5 minutes, students may work with their partner or in their small groups. Support students problem-solving by:

- asking questions related to the mathematical ideas, problem-solving strategies, and connections between representations.
- asking students to explain their thinking and reasoning.

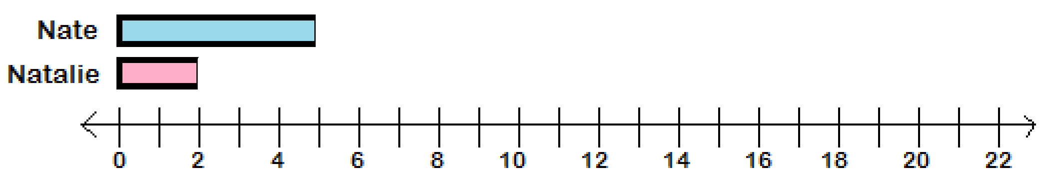
TASK DIRECTIONS:

Nate and his sister Natalie are walking around the track at school and they're walking at a steady rate. Nate walks 5 feet in 2 seconds while Natalie walks 2 feet in the same amount of time.

a) *Use the Cuisenaire rods to represent Nate and Natalie's walk around the track.*
Suggestion: ***Noticing Nate and Natalie walk the same amount of time, students will compare the feet walked within the given time. Students can use 5 white rods to represent Nate and 2 white rods to represent Natalie. They could also use any rods which show a 5:2 ratio.***

b) Draw a diagram or picture that represents Nate and Natalie's walk around the track.

Solution:
Students may use a variety of representations. For example, bar models or a number line as shown below could be used to represent the walk around the track.



Questions to encourage thinking:

- ***Are Nate and Natalie walking at the same rate?***
- ***Can you explain what your diagram shows about Nate and Natalie's walk?***

c) Set up a table to represent this situation. Let the x-axis represent the number of feet that Nate walks and the y-axis represents the number of feet Natalie walks.

Solution:

Nate (x)	*Natalie (y)*
0	*0*
5	*2*

10	***4***
15	***6***
20	***8***

d) What patterns do you see in the table? Explain the pattern. Express this as an equation.

Solution:
Students should be able to explain that for every 5 steps Nate makes, Natalie makes 2. So the ratio of Nate's steps to Natalie's is 5:2. The equation that represents the situation is $y = x/2.5$

e) When Nate walks 45 feet, how far will Natalie walk? Explain in writing or show how you found your answer.

Solution:
Students may use the equation, table, or graph as a way to answer the problem. They may also set up a proportion to solve the problem. For example, $\frac{5}{2} = \frac{45}{x}$. When solved the answer is 18. Natalie will have taken 18 steps when Nate has taken 45 steps.

Comment:
For further discussion, ask, "Can you predict how far Natalie will walk if Nate walks 1000 feet?" This discussion should focus on the most efficient methods for solving the problem (equation or proportion) versus using a table or a graph.

Nate & Natalie's Walk

Nate and his sister Natalie are walking around the track at school and they're walking at a steady rate. Nate walks 5 feet in 2 seconds while Natalie walks 2 feet in the same amount of time.

a) *Use the Cuisenaire rods to represent Nate and Natalie's walk around the track.*

b) Draw a diagram or picture that represents Nate and Natalie's walk around the track.

c) Set up a table to represent this situation. Let the x-axis represent the number of feet that Nate walks and the y-axis represents the number of feet Natalie walks.

d) What patterns do you see in the table? Explain the pattern. Express this as an equation.

e) When Nate walks 45 feet, how far will Natalie walk? Explain in writing or show how you found your answer.

Buses

Back to task table

In this task, students will work with a distance-time graph to describe a bus journey.
Source: Balanced Assessment Materials from Mathematics Assessment Project
http://www.map.mathshell.org/materials/download.php?fileid=1070

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.RP.2 Recognize and represent proportional relationships between quantities.

MGSE7.RP.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

MGSE7.RP.2c Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.

MGSE7.RP.2d Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1,r)$ where r is the unit rate.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.

ESSENTIAL QUESTION

How do I interpret a distance time graph and determine a point of intersection?

TASK COMMENTS

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=summative

The task, *Buses*, is a Mathematics Assessment Project Assessment Task that can be found at the website: http://www.map.mathshell.org/materials/tasks.php?taskid=365&subpage=apprentice

The PDF version of the task can be found at the link below:
http://www.map.mathshell.org/materials/download.php?fileid=1070

The scoring rubric can be found at the following link:
http://www.map.mathshell.org/materials/download.php?fileid=1071

Sphero Draw & Drive

[Back to task table]

Task adapted from http://mikewiernicki.com

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.RP.2 Recognize and represent proportional relationships between quantities.

MGSE7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, and fees.

MGSE7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

ESSENTIAL QUESTIONS

- How are proportional relationships used to solve multistep ratio and percent problems?
- How do equations represent proportional relationships?

MATERIALS REQUIRED

- 3-Act task videos http://mikewiernicki3act.wordpress.com/sphero-draw-drive/
- Adding machine tape for tape diagrams (one option for students)

TEACHER NOTES

In this task, students will watch the video, then tell what they noticed. They will then be asked to discuss what they wonder or are curious about. These questions will be recorded on a class chart or on the board. Students will then use mathematics to answer their own questions. Students will be given information to solve the problem based on need. When they realize they don't have the information they need, and ask for it, it will be given to them.

TASK DESCRIPTION

The following 3-Act Task can be found at: http://mikewiernicki3act.wordpress.com/sphero-draw-drive/

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

ACT 1:

Watch the video: http://vimeo.com/94466728

Ask students what they noticed and what they wonder. Record responses.
Suggested question: How far did the ball roll? How much farther does the ball need to roll in order to complete the rectangle?
Estimate. Write an estimate that is too high and an estimate that is too low.

ACT 2:

The following information is provided for students as they ask for it.

http://vimeo.com/94422438

ACT 3

Students will compare and share solution strategies.

- Reveal the answer. Discuss the theoretical math versus the practical outcome.
- How appropriate was your initial estimate?
- Share student solution paths. Start with most common strategy.
- Revisit any initial student questions that weren't answered.

Sphero Draw and Drive Name:________________________

Adapted from Andrew Stadel

ACT 1

What did/do you notice?

What questions come to your mind?

Main Question:____

Estimate the result of the main question? Explain?
Place an estimate that is too high and too low on the number line
Low estimate *Place an "x" where your estimate belongs* High estimate

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimate using this information:_______________________________

Act 2 (con't)
Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.

Comparing Strategies for Proportion Problems - (FAL)

Back to task table

(Content Development)
This lesson unit is intended to help you assess whether students recognize relationships of direct proportion and how well they solve problems that involve proportional reasoning.

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=1306

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction (1/2)/(1/4) miles per hour, equivalently 2 miles per hour.

MGSE7.RP.2 Recognize and represent proportional relationships between quantities.

MGSE7.RP.2a Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

MGSE7.RP.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

MGSE7.RP.2c Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.

MGSE7.RP.2d Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1,r)$ where r is the unit rate.

MGSE7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, and fees.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
8. Look for and express regularity in repeated reasoning.

ESSENTIAL QUESTIONS

- How do I solve problems involving proportional reasoning?
- How do I identify relationships that are directly proportional?

TASK COMMENTS

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Developing a Sense of Scale*, is a Formative Assessment Lesson (FAL) that can be found at the website:
http://map.mathshell.org/materials/lessons.php?taskid=456&subpage=problem

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=1306

Creating A Scale Map

[Back to task table]

Adapted from Muschia, G. R. &., Muschia, J.A. (1996). Hands-on math projects with real-life applications. (pp. 112-115). New Jersey: Jossey-Bass.

Students have many opportunities to copy and read maps, however not to create one. When students have to create a map, they realize that mathematics plays a major role in map making. ***S****tudents can work in pairs or groups of 3. Students will create a scale map of their school, school grounds, or their yard at home. They will include landmarks, important details, legends, and an accurate scale.*

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction (1/2)/(1/4) miles per hour, equivalently 2 miles per hour.

MGSE7.RP.2 Recognize and represent proportional relationships between quantities.

MGSE7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

STANDARDS FOR MATHEMATICAL PRACTICES

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
5. Use appropriate tools strategically.
6. Attend to precision.

COMMON MISCONCEPTIONS

A common error is to reverse the position of the variables when writing equations. Students may find it useful to use variables specifically related to the quantities rather than using x and y.

Constructing verbal models can also be helpful. They can use this verbal model to construct the equation. Students can check their equation by substituting values and comparing their results to the table. The checking process helps student revise and recheck their model as necessary.

ESSENTIAL QUESTIONS:

- How are distances and measurements translated into a map or scale drawing?
- How do I determine the an appropriate scale for the area (such as my yard or school) that I am measuring and mapping?

MATERIALS

- Student Guide 3-1
- Student Guide 3-2

- Measuring tapes
- Yardsticks
- Meter sticks; or trundle wheels;
- compasses (for finding directions)
- rulers
- crayons or colored pencils or markers
- poster paper (approximately 22 x 28 inches).

GROUPING

Individual / Partner or groups of 3

TASK DESCRIPTIONS

Give students options regarding the maps they would like to draw. Students who have a large yard may wish to measure and create a map of that; students who live in apartments might prefer to measure and create a map of their school or school grounds. If students are to draw a map of their school, you might decide to limit the map to a section or wing.

- Begin this project by explaining that maps use a scale to ensure accurate distances. On a world map, one inch might equal 1,500 miles, while on the map of a small town, an inch might equal one-fourth of a mile. On a map of a yard or block, an inch might only equal several feet. Scales are created with mathematics.

- Distribute copies of Student Guide 3-1 and review it with your students.

Creating a Scale Map

- If you wish, you may simplify the project by instructing students to create a scale map of your school or school grounds. In this way, you may take your class to obtain the necessary measurements together.

- In measuring distances, remind students to be as accurate as possible. We suggest that distances be rounded to the nearest foot, nearest yard, or nearest meter.

- Suggest that students sketch a rough copy of their map first. This will help them to “see” relationships and estimate where things should be.

- Distribute copies of Student Guide 3-2, “How to Make a Map.” Go over it with your students, making sure they understand how to create a scale for their maps. You may want to check the scales students select before they start making the final copies of their maps.

- After students draw their maps, they should label and color them. Suggest that they provide legends, label the direction, and, of course, note the scale.

DIFFERENTIATION

Extension

Encourage students to pursue a study of an instrument called the architect's scale, and use it to make a scale drawing of their house. If their house has two stories, they may need to create more than one drawing.

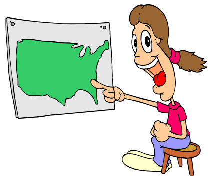
Intervention

Select the scales that the students will use for their maps.

Solution:

Solutions to student's maps will vary. To assess student understanding, make sure that measured distances are reasonable and that students have an accurate and appropriate scale for their drawing.

Student Guide 3-1 Name: ________________________________



Creating A Scale Map

Situation/Problem

You and your partner(s) are to create a scale map of a familiar place such as your school, school grounds, or the yard of your home.

Possible Strategies

1. Accurately measure distances (rounding to the nearest foot, yard, or meter).
2. Note landmarks. In case of a yard, this might include things like trees, woodpiles, sheds, etc. You might include such things in a legend on your map.
3. Create a rough sketch of your map before drawing a final copy. A "rough sketch" will help you to visualize perspectives and landmarks.

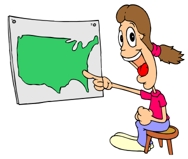
Special Considerations

- Use a measuring tape, yardstick, meter stick or trundle wheel for measuring distances.
- Use a pad and pencil to record distances. Don't try to remember the distances; this may cause mistakes in your map and your scale.
- As you record distances, sketch your map, placing landmarks "about" where they would be. Record the distances in feet, yards, or meters. It's a good idea to locate landmarks using the measurements from two boundaries.
- Use a compass to find directions. Be sure to label the directions correctly on your map.
- Consult Student Guide 3-2 for information about working with scale drawings.
- Be sure the final copy of your map is accurate. Label distances and landmarks, add color, and include a legend and directions. You may want to compare your map to the original area and check it for accuracy.

To Be Submitted

1. Your scale map
2. Your records of measurements and calculations

Student Guide 3-2 Name: ______________________________



How To Make A Map

1. Decide upon the boundaries of your map.
2. Make a sketch of the area you will include on your map. Note the approximate position of any landmarks. In a school, land marks might include stairwells, display cases, or water fountains. Landmarks in a yard might include trees, flowerbeds, decks, sheds, or woodpiles.
3. Accurately measure the boundaries (length and width) of the area. Locate the position of landmarks by obtaining at least two measurements from boundaries.
4. Select the scale by considering your longest measurement, and how to "fit" it on the paper. Remember that the scale should be as long as possible so that your map will look good on the paper.
5. To choose the best scale, divide the longest length of your paper in inches (or centimeters) by the longest dimension of the boundary in feet (or meters).
   - Round your quotient down to the nearest quarter or eighth inch (or centimeter). Here's an example: The longest boundary (longest length) on your map is 80 feet. The longest dimension of your paper is 28 inches. $\frac{28}{80} = .35$. Since .35 is between .25 (one fourth inch) and 0.375 (three-eighths inch), you must round down so that your scale will be $\frac{1}{4}$ inch = 1 foot.
   - Now take the other dimension of the boundary and the other dimension of the paper, and divide the length of the paper by the length of the boundary.
   - Round your quotient down to the nearest quarter or eighth inch (or centimeter).
   - Compare the scales. If they are the same, great! If they are different, use the smaller scale.

6. To place items on your map, use your measurements and the scale you have chosen. For example, suppose an apple tree is 21 feet from the fence on the eastern side of the yard, and 16 feet from the fence on the northern side. If your scale is $\frac{1}{4}$ inch = 1 foot, multiply the number of inches by the number of feet to determine the number of inches the actual distance would be on your map. Note the example of the math below.

$$\frac{1}{4} \times \frac{21}{1} = \frac{21}{4} = 5\frac{1}{4}$$

$$\frac{1}{4} \times \frac{16}{1} = 4$$

Place the tree $5\frac{1}{4}$ inches from the fence on the eastern side, and 4 inches from the northern side.

Drawing to Scale: Designing a Garden – (FAL)

(Problem Solving Task) **Back to task table**
This lesson is intended to help assess how well students are able to interpret and use scale drawings to plan a garden layout. This lesson also addresses students' ability to apply proportional reasoning and metric units.
Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=1376

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction (1/2)/(1/4) miles per hour, equivalently 2 miles per hour.

MGSE7.RP.2 Recognize and represent proportional relationships between quantities.

MGSE7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically
6. Attend to precision.

ESSENTIAL QUESTIONS

How can proportional relationships be analyzed to determine the reasonableness of the scale factor?
How are geometrical figures constructed and used to analyze the relationships between figures?
How are real-life mathematical problems solved using algebraic equations?

TASK COMMENTS

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:

http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Drawing to Scale: Designing a Garden*, is a Formative Assessment Lesson (FAL) that can be found at the website:
http://map.mathshell.org/materials/lessons.php?taskid=494&subpage=problem

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:

http://map.mathshell.org/materials/download.php?fileid=1376

Fish In A Lake

Back to task table

Source: Teaching Student Centered Mathematics, vol. 3 Grades 5-8, by John A. Van de Walle

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.RP.2 Recognize and represent proportional relationships between quantities.

MGSE7.RP.2a Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

MGSE7.RP.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

MGSE7.RP.2c Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.

MGSE7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, and fees.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.

ESSENTIAL QUESTIONS

- How is the unit rate represented in tables, graphs, equations and diagrams?
- How is unit rate computed in real-world problems?
- How are ratios and their relationships used to solve real world problems?

MATERIALS REQUIRED

- Paper bags or small boxes
- Counters or small cubes
- Small stickers to mark the cubes or counters

TEACHER NOTES

Proportional relationships can be confusing to students. In order for students to make sense of proportional relationships and develop strategies, students need a context. The context gives students a starting point to make sense of the problems and develop strategies for solving them.

Solving problems in a different context from money, helps students develop a deeper understanding with ratios and proportions.

This task embeds data collection and estimation into ratio and proportional reasoning. Teachers may want students to create their own "lakes" of "fish" in this task by having them randomly dump counters into paper bags (or shoe boxes). This is the only set-up needed for this task and can be accomplished very quickly.

While this task can be done individually, the benefits of using partners or small groups to promote discussion of mathematical ideas and strategies may outweigh any need for an individual grade. As a teacher, listening to students' discussions of mathematical ideas and strategies can be extremely informative and valuable. This data can be used to inform instruction, decide next steps for individual or groups of students, or as a formative assessment.

TASK DIRECTIONS:

- Prepare a paper bag or shoebox full of some uniform small object such as centimeter cubes or plastic chips. If the box is your lake and the objects are the fish you want to count, how can you estimate the number without actually counting them? Remember, if they were actual fish, you couldn't even see them!
- Have a student reach into the box and "capture" a representative sample of the "fish." Tag each fish by marking it in some way – small stickers work well.
- Count and record the number tagged and then return them to the box.
- Put the stickers away. You will not be tagging any more fish!
- The assumption of the scientist is that tagged animals will mix uniformly with the larger population, so mix them thoroughly. Next, make a recapture of fish from the box.
- Count the total captured and the number in the capture that are tagged. Release the "fish" back into the "lake." Do this several times. Accumulate these data.
- Now the task is to use all of the information to estimate the number of fish in the lake. The recapture data provide an estimated ratio of tagged to untagged fish. The number tagged to the total population should be in the same ratio. After solving the proportion, have students count the actual items in the box to see how close their estimate is.

DIFFERENTIATION

Extension:
Students needing extensions may wish to develop models for other situations similar to this task.

Intervention:
Students needing support might try using a larger tagged sample. This, and support from the teacher may give students the support they need for success with this task.

Fish In A Lake

1. Prepare a paper bag or shoebox full of some uniform small object such as centimeter cubes or plastic chips. If the box is your lake and the objects are the fish you want to count, how can you estimate the number without actually counting them? Remember, if they were actual fish, you couldn't even see them!

2. Have a student reach into the box and "capture" a representative sample of the "fish." Tag each fish by marking it in some way – small stickers work well.

3. Count and record the number tagged and then return them to the box.

4. Put the stickers away. You will not be tagging any more fish!

5. The assumption of the scientist is that tagged animals will mix uniformly with the larger population, so mix them thoroughly. Next, make a recapture of fish from the box.

6. Count the total captured and the number in the capture that are tagged. Release the "fish" back into the "lake." Do this several times. Accumulate these data.

7. Now the task is to use all of the information to estimate the number of fish in the lake. The recapture data provide an estimated ratio of tagged to untagged fish. The number tagged to the total population should be in the same ratio.

8. Check to see how close your estimate of how many total fish are in the pond by counting them.

Patterns & Percentages

Back to task table

The learning task is focused on using models and patterns to develop number sense for solving percent problems. The lesson will link models, tables, graphs and proportional reasoning to percent problems.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.RP.2 Recognize and represent proportional relationships between quantities.

MGSE7.RP.2a Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

MGSE7.RP.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

MGSE7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, and fees.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

ESSENTIAL QUESTIONS

- How can models be used to solve percent problems?
- How do I apply mental math strategies to solve percent problems?

MATERIALS

- Warm-up Problems- Models & Patterns
- Patterns & Percentages Task sheet

GROUPING

Individual / Partner

TASK DESCRIPTION

Comments

Percent problems are specific kinds of proportion problems. Students began their work with percent problems in 6th Grade Math. 7th Grade Math expands their understanding and fluidity with percent problems. Students should be reminded that a percent is a part-to-whole relationship wherein the whole is always 100.

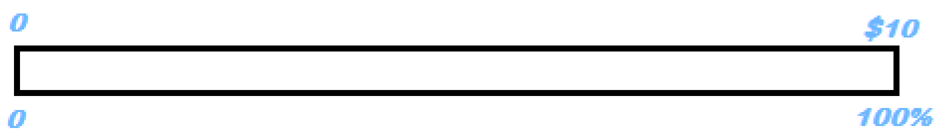
Models are excellent tools to help students connect concrete to algebraic representations. **Use the following mini-lesson to introduce the task.**
Ask students, how would you write 10% as a decimal? Ask a volunteer to record it on the board.
Ask students, how would you write 10% as a fraction? Ask a volunteer to record it on the board.

Draw the bar below on the board. Ask students, if the bar below represents a whole number amount, then what percent would it equal?

***Solution*:**

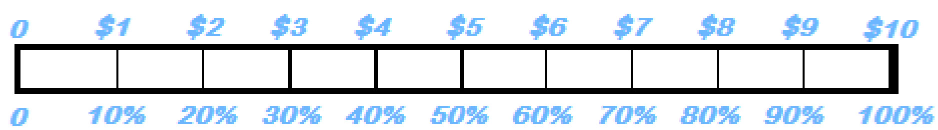
100%

Now let's give the bar a value. The bar now represents $10 which is 100%. Label the bar.



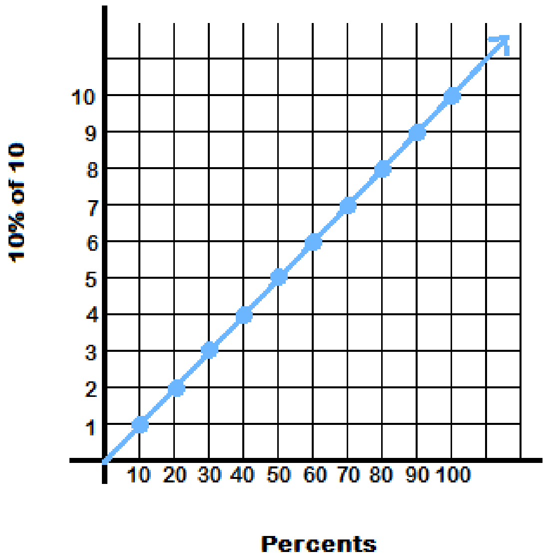
Solution:
Solutions are in blue on the bar model.

Ask a student to divide the bar up into ten equal parts. Label each part with its correct percent value and the correct money value.



Solution:
Solutions are in blue on the bar model.

Let's look at this relationship. Is this a proportional relationship? How do you know?
With a partner, ask students to fill in the following table and graph the relationship. Encourage them to use the values from the bar model above.
Solution:
Solutions are in blue in the table and graph below. The relationship of 10% of 10 is a proportional relationship. You may show students the relationship as a proportion. For example, $\frac{10}{100} = \frac{1}{10}$. We usually have students set up the proportion as $\frac{Percent}{100} = \frac{part}{whole}$.



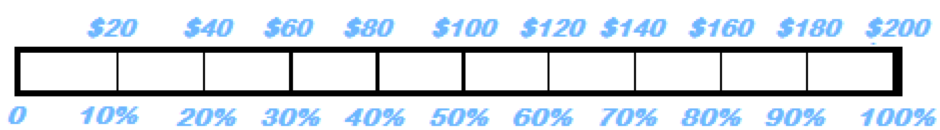
Percent	Amount based on $10
0	***0***
10	***1***
20	***2***
30	***3***
40	***4***
50	***5***
60	***6***
70	***7***
80	***8***
90	***9***
100	***10***

Emphasize the application of mental math strategies. This may help students determine whether or not their answers to problems are reasonable.

I. More Modeling and making use of structure

For problems 1 – 5, label the percent bar with its appropriate dollar values and percent values. Divide the bar into ten equal parts. Then find 10% of the total. Fill in the pieces of the proportion to represent the relationship.

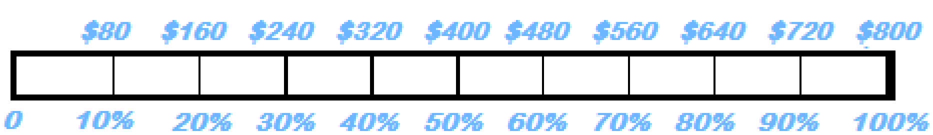
1] The total amount is $200.



Solution:

Solutions are in blue on the bar model. $\frac{10\%}{100\%} = \frac{20}{200}$

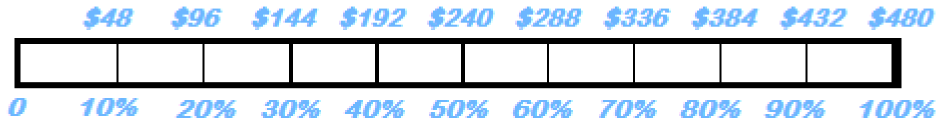
2] The total amount is $800.



Solution:

Solutions are in blue on the bar model. $\frac{10\%}{100\%} = \frac{80}{800}$

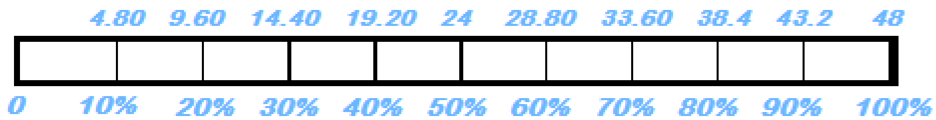
3] The total amount is $480.



Solution:

Solutions are in blue on the bar model. $\frac{10\%}{100\%} = \frac{48}{480}$

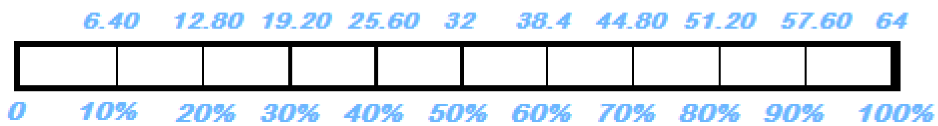
4] The total amount is $48.



Solution:

Solutions are in blue on the bar model. $\frac{10\%}{100\%} = \frac{4.80}{48}$

5] The total amount is $64.



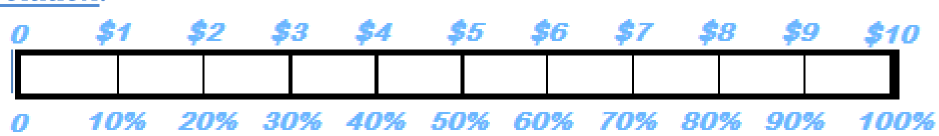
Solution:

Solutions are in blue on the bar model. $\frac{10\%}{100\%} = \frac{6.40}{64}$

II. **Make sense of problems and persevere in solving them:** Answer the following problems based on what we have learned about percents and patterns. You may use the model, table or graph above to help you. (Hint: Use what you know about 10% of 10 to help you answer each problem.)

1. Mikayla went to the mall to do some shopping. The sign in her favorite store's window read, *Big Sale-20% off of everything in the store*. Mikayla bought headphones for her I-Pod that were regularly priced $10. Draw a percent bar to show the price of the headphones, the 20% discount, and shade in the part of the bar representing how much Mikayla actually paid for the headphones (before tax).

Solution:



Students should shade in the first 8 boxes on their bar.

2. Write a proportion that would help you determine what 20% of \$10 is and then show the next steps in finding the price of the headphones (before tax).

   Solution:
   \$8.00: Use the graph or bar model to help students see 20% of \$10. Discuss discounts. Students can use many different forms of the proportion shown. They can use cross multiplication and a variable. Answers should include showing how students arrived at \$8 through subtraction if they are not able to use proportions to figure out the \$8 directly.

$$\frac{20\%}{100\%} = \frac{\$8}{\$10}$$

3. The county that Mikayla's lives in charges 5% sales tax on all purchases. What is the final price Mikayla will pay for her headphones after the discount and tax have been applied? Use a percent bar, proportion, or any other method to help you solve this problem.
   Solution: (\$8)(.05)=\$.40 \$8.40
4. Mikayla wants to buy a pair of jeans that is on sale for 25% off. The original price was \$64 and sales tax is 5%. What is the total price the cashier at the store will ask Mikayla to pay for the jeans?
   Solution: (\$64)(.25)=\$16.00 \$64-\$16=\$48 (\$48)(.05)=\$2.40
   \$48+\$2.40=\$50.40

5. Saira needs to purchase a dress, 2 pairs of shoes, and a tiara for her quinceañera. The dress costs \$212, both pairs of shoes cost \$38, and the tiara is \$18. She pays 5% sales tax. Show whether or not she pays more, less, or the same amount of sales tax if she purchases the items all together in one transaction, or buys each item separately from different stores.
   Solution: (\$212)(.05)=\$10.60 \$222.60
   (\$38)(2)(.05)=\$3.80 \$79.80
   (\$18)(.05)=\$.90 \$18.90

   Total= \$321.30 Total Tax on \$306=\$15.30 She will pay the same amount of tax if she shops at one store, or multiple stores buying each item separately. Explanations will vary.
6. José referees soccer games on the weekend. He makes \$12 per hour when working a recreational league game, and earns 15% extra when he refs for a travel team game. If soccer games are typically an hour and a half long, how much would he get paid for refereeing 2 recreational games followed by 2 travel team games?

   Solution:3 hours rec games=\$12/hour x 3 hours=\$36

New unit rate for travel team games: \$12/hour x .15= \$1.80
\$12.00+\$1.80=\$13.80/hour

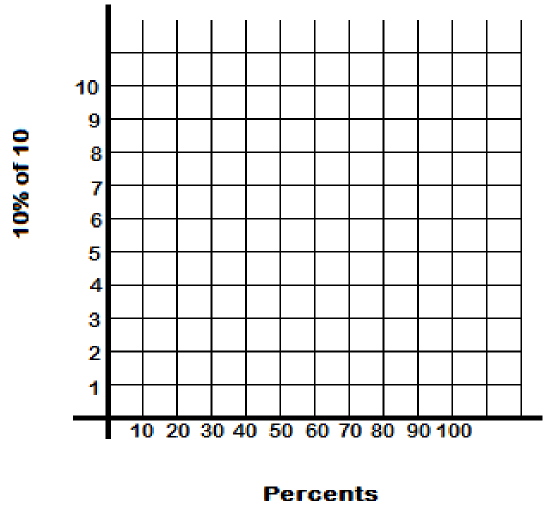
3 hours travel games=\$13.80 x 3 hours= \$41.40

Total \$ for all games worked: \$36.00+\$41.40=\$77.40

<u>Patterns & Percentages</u>

Let's look at this relationship. Is this a proportional relationship? How do you know?
Fill in the following table and graph the relationship. We will use the values from the bar model above.

Percent	Amount based on $10



I. More Modeling and making use of structure

For problems 1 – 5, label the percent bar with its appropriate dollar values and percent values. Divide the bar into ten equal parts. Then find 10% of the total. Fill in the pieces of the proportion to represent the relationship.

1] The total amount is $200.

$$\frac{10\%}{100\%} = \frac{\$20}{\$200}$$

2] The total amount is $800.

$$\frac{10\%}{100\%} = \frac{\$}{\$}$$

3] The total amount is $480.

4] The total amount is $48.

5] The total amount is $64.

1. Mikayla went to the mall to do some shopping. The sign in her favorite store's window read, *Big Sale-20% off of everything in the store*. Mikayla bought headphones for her I-Pod that were regularly priced $10. Draw a percent bar to show the price of the headphones, the 20% discount, and shade in the part of the bar representing how much Mikayla actually paid for the headphones (before tax).

2. Write a proportion that would help you determine what 20% of $10 is and then show the next steps in finding the price of the headphones (before tax).

3. The county that Mikayla's lives in charges 5% sales tax on all purchases. What is the final price Mikayla will pay for her headphones after the discount and tax have been applied? Use a percent bar, proportion, or any other method to help you solve this problem.

4. Mikayla wants to buy a dress that is on sale for 25% off. The original price was $84 and sales tax is 5%. What is the total price the cashier at the store will ask Mikayla to pay for the dress?

5. Saira needs to purchase a dress, 2 pairs of shoes, and a tiara for her quinceañera. The dress costs $212, both pairs of shoes cost $38, and the tiara is $18. She pays 5% sales tax. Show whether or not she pays more, less, or the same amount of sales tax if she purchases the items all together in one transaction, or buys each item separately from different stores.

6. José referees soccer games on the weekend. He makes $12 per hour when working a recreational league game, and earns 15% extra when he refs for a travel team game. If soccer games are typically an hour and a half long, how much would he get paid for refereeing 2 recreational games followed by 2 travel team games?

<u>Increasing and Decreasing Quantities by a Percent</u> – (FAL)

Back to task table

(Concept Development)
This lesson unit is intended to help you assess how well students are able to interpret percent increase and decrease.

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/lessons.php?unit=7100&collection=8

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction (1/2)/(1/4) miles per hour, equivalently 2 miles per hour.

MGSE7.RP.2 Recognize and represent proportional relationships between quantities.

MGSE7.RP.2a Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

MGSE7.RP.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

MGSE7.RP.2c Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.

MGSE7.RP.2d Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1,r)$ where r is the unit rate.

MGSE7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, and fees.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

ESSENTIAL QUESTIONS

- How do I utilize percent of increase and decrease as an aspect of multiplication?

TASK COMMENTS

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Increasing and Decreasing Quantities by a Percent*, is a Formative Assessment Lesson (FAL) that can be found at the website:
http://map.mathshell.org/lessons.php?unit=7100&collection=8

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=1249

25% Sale

Back to task table

In a sale, the store reduces all prices by 25% each week. Does this mean that, after 4 weeks, everything in the store will cost $0? If not, why not?

Source: Balanced Assessment Materials from Mathematics Assessment Project
http://www.map.mathshell.org/materials/download.php?fileid=1042

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction (1/2)/(1/4) miles per hour, equivalently 2 miles per hour.

MGSE7.RP.2 Recognize and represent proportional relationships between quantities.

MGSE7.RP.2a Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

MGSE7.RP.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

MGSE7.RP.2c Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.

MGSE7.RP.2d Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1,r)$ where r is the unit rate.

MGSE7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, and fees.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

ESSENTIAL QUESTION

How do I solve and interpret solutions of real-world percent problems?

TASK COMMENTS:

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=summative

The task, *25% Sale*, is a Mathematics Assessment Project Assessment Task that can be found at the website: http://www.map.mathshell.org/materials/tasks.php?taskid=358&subpage=apprentice

The PDF version of the task can be found at the link below:
http://www.map.mathshell.org/materials/download.php?fileid=1042

The scoring rubric can be found at the following link:
http://www.map.mathshell.org/materials/download.php?fileid=1043

Ice Cream

Back to task table

Source: Balanced Assessment Materials from Mathematics Assessment Project
http://www.map.mathshell.org/materials/download.php?fileid=1157

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction (1/2)/(1/4) miles per hour, equivalently 2 miles per hour.

MGSE7.RP.2 Recognize and represent proportional relationships between quantities.

MGSE7.RP.2a Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

MGSE7.RP.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

MGSE7.RP.2c Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.

MGSE7.RP.2d Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1,r)$ where r is the unit rate.

MGSE7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, and fees.

STANDARDS FOR MATHEMATICAL PRACTICE

This task uses all of the practices:
1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

ESSENTIAL QUESTION

How do I analyze and solve proportional relationships in real-world contexts?

TASK COMMENTS

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=summative

The task, *Ice Cream*, is a Mathematics Assessment Project Assessment Task that can be found at the website: http://www.map.mathshell.org/materials/tasks.php?taskid=389&subpage=expert

The PDF version of the task can be found at the link below:
http://www.map.mathshell.org/materials/download.php?fileid=1157

The scoring rubric can be found at the following link:
http://www.map.mathshell.org/materials/download.php?fileid=1158

Which is the Better Deal?

Back to table

Adapted from Muschia, G. R. &., Muschia, J.A. (1996). Hands-on math projects with real-life applications. (pp. 126-130). New Jersey: Jossey-Bass.

Students will evaluate and compare the packaging and prices of products they select from advertisements or the internet. For example, if a student selects canned colas, they will compare the price of a12-pack of canned coke to a 6-pack of canned coke and determine which buy would be the better deal. Working individually or in pairs, students will select three different products.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction (1/2)/(1/4) miles per hour, equivalently 2 miles per hour.

MGSE7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, and fees.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.

ESSENTIAL QUESTIONS

- How does my understanding of unit rate save me money?
- How can I determine the unit rate for a product that I might purchase?

MATERIALS

- Student Guide 3-5
- Student Data Sheet 3-6
- White poster paper
- Rulers
- markers and felt-tipped pens for making charts
- Samples of products (Students can clip advertisements or print products off the internet. They need to have a picture of the product, explanation/details, price)

GROUPING

Individual / group

TASK DESCRIPTION

Prior to the task, the teacher will introduce the relationship between unit rate and purchasing products.

We live in a society where we have a choice of purchasing countless consumer products. Just consider how many types of sneakers we can buy, or how many brands of potato chips we can choose from. To help us select the best package of products, we are going to look at how to figure unit price.

Ask your students what rationale they use to buy products. For example, why did they buy the drinks they have at home? Most likely they buy the same type drinks every time. They either purchase the same drinks due to price, taste, or some other reason. Explain that people usually have reasons for buying one product instead of another.

- Begin the project by explaining that students will select a type of product – see Student Data Sheet 3-6 for some examples.
- Distribute copies of Student Guide 3-5 and review it with your students. Emphasize that they should compare three different products and each product needs to be packaged in three different sizes. For example, if the student chooses coke then they need to price it as a 6-pack, 12-pack, and 24-pack (and each package needs to have the same number of ounces). Teachers can model this with bringing in examples of products with the same ounces or use ads.
- Remind students to create a rough chart on scrap paper before attempting to draw their final copy. Encourage them to design a chart that will be informative as well as easy to read.

As a wrap-up to the task, students may present oral presentations using charts and /or other evidence to support their findings. You may also wish to display the charts.

Solution:
Key elements to look for in the student's work are: (1) accurate calculations of unit rate for the products selected, (2) reasonableness in their comparisons and unit rate, and (3) correct use of terminology when orally presenting their findings.

Student Guide 3-5 Name: ______________________________

Which Is The Better Deal?

Situation/Problem

You and your partner(s) are to select a product, and compare the size of package and price (three different sizes/prices). You are to trying to determine which is the best deal by finding their unit price. After you have reached your conclusions, design a chart to support your findings and present your data to the class through an oral report.

Possible Strategies

1. Look in sales papers for groceries/retail stores.
2. Brainstorm with your partner(s) which products you might like to compare.

Special Considerations

- After selecting your product, decide which size or quantity in a package you will compare. Write these categories on a sheet of paper, then compare the price.
- After obtaining your data, analyze it and make decisions comparing quantity/size to price. Compute the unit rate.
- Create a chart illustrating your results. Sketch a rough copy of your chart first. This enables you to revise the chart before starting the final copy. Arrange the design so it presents the data clearly. List your products by brand name and show your comparison of quantity/size to price. If there is room on your chart, you may wish to provide a brief summary of your results and why you chose that quantity/size product for that price.
- Before presenting your findings to the class, write notes so that you don't forget to mention any important information. Rehearse your presentation.

To Be Submitted

1. Research/Comparison Notes
2. Chart

Student Data Sheet 3-6 Name: ________________________________

Which Is The Better Deal

Popular products are compared regularly. Many educated consumers rely on unit pricing to make sure they are getting the best deal to fit their needs and budgets.

Some products and quantity/size to compare:

- ✓ Soda
- ✓ Potato Chips
- ✓ Ice Cream
- ✓ Milk
- ✓ Paper products
- ✓ Snack crackers

Any product that is packaged in more than one size can be compared. For example, you could compare the unit price of a 6-pack of Coke to the unit price of a 6-pack of Pepsi.

Technology Resources

Back to task table

https://www.illustrativemathematics.org/7.RP

MGSE7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction (1/2)/(1/4) miles per hour, equivalently 2 miles per hour.

http://nzmaths.co.nz/resource/comparing-finding-rates

http://nzmaths.co.nz/resource/ratios-and-rates

http://nzmaths.co.nz/resource/playing-money-other-point-view

http://mathmistakes.org/category/grade-7/ratios-proportional-relationships-grade-7/

http://www.estimation180.com/lessons.html

MGSE7.RP.2 Recognize and represent proportional relationships between quantities.

http://nzmaths.co.nz/resource/comparing-finding-rates

http://nzmaths.co.nz/resource/ratios-and-rates

http://nzmaths.co.nz/resource/ratios

http://nzmaths.co.nz/resource/playing-money-other-point-view

http://mathmistakes.org/category/grade-7/ratios-proportional-relationships-grade-7/

MGSE7.RP.2a Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

http://nzmaths.co.nz/resource/comparing-finding-rates

http://www.estimation180.com/lessons.html

MGSE7.RP.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

MGSE7.RP.2c Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.

http://nzmaths.co.nz/resource/ratios-and-rates

MGSE7.RP.2d Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1, r)$ where r is the unit rate.

MGSE7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, and fees.
http://nzmaths.co.nz/resource/comparing-finding-rates
http://nzmaths.co.nz/resource/body-ratios
http://illuminations.nctm.org/Lesson.aspx?id=1049
http://nzmaths.co.nz/resource/playing-money-other-point-view
http://nzmaths.co.nz/resource/percentages
http://nzmaths.co.nz/resource/percentages-problems-two-steps
http://nzmaths.co.nz/resource/estimating-percentages
http://nzmaths.co.nz/resource/percentage-bar
http://nzmaths.co.nz/resource/getting-percentible
https://sites.google.com/site/sensiblemathematics/activities-by-concepts/percentages

MGSE7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

http://nzmaths.co.nz/resource/scale-factors-areas-and-volumes
http://nzmaths.co.nz/resource/russian-boxes
https://www.illustrativemathematics.org/content-standards/7/G/A/1/tasks
http://illuminations.nctm.org/Lesson.aspx?id=1049