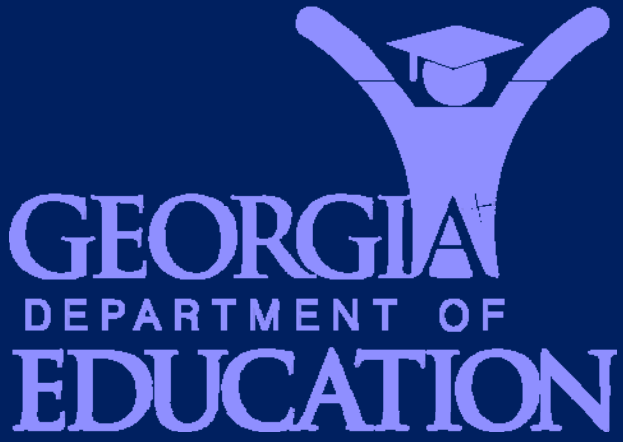
The Georgia Performance Standards for K-12 Mathematics



Kathy Cox

State Superintendent of Schools

Accelerated Mathematics I Training

May and June 2008

Overview of Day 1

- **Big Picture of the New Curriculum**
- **Climbing the Ladder**
- **6^{th}, 7^{th}, 8^{th} Grade and Mathematics I Tasks**

Group Norms and Housekeeping

Group Norms:

- Ask questions
- Work toward solutions
- Honor confidentiality
- Meet commitments or let others know if you are struggling

Housekeeping:

- Parking Lot
- Phone calls
- Restrooms
- Breaks
- Lunch

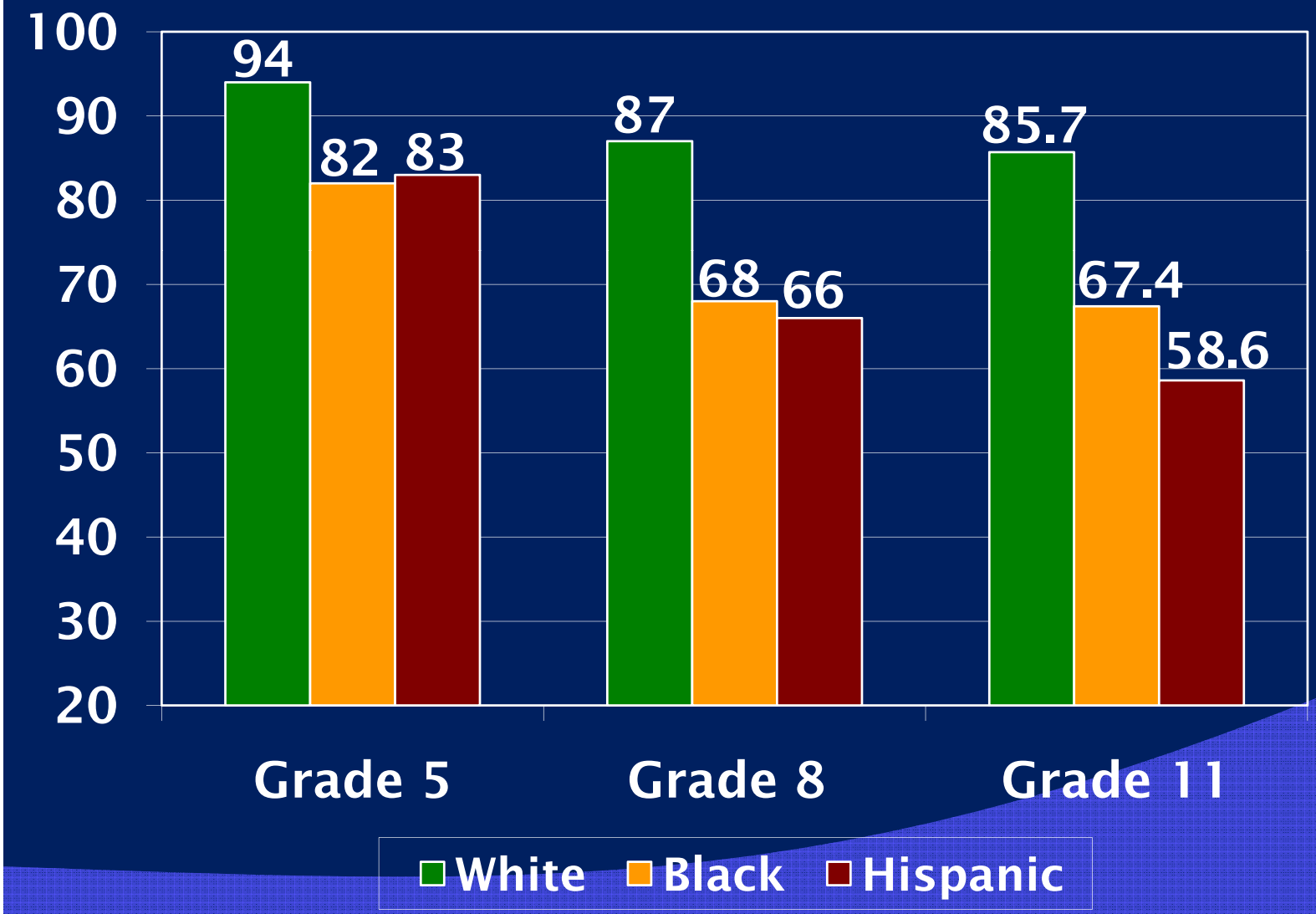
The Essential Questions

- Why a new curriculum?
- What process was used in the development of the new standards?
- What is the new curriculum? How is it different?

WHY a new curriculum?

- 2001 PDK audit of Quality Core Curriculum
  - Lacked rigor and depth-“A mile wide and an inch deep”
  - Did not allow for the alignment of instruction and assessment
  - Did not provide clear expectations for students
- Age of QCC-written in 1985, one cursory revision in 1997
- Not aligned to national and international standards

Georgia's Student Achievement in Math



Georgia's Student Achievement in Mathematics

- Percent of SAT Test Takers w/ 4 years of Mathematics
  - NATION: 62 percent
  - GEORGIA: 69 percent
- Score for SAT Test Takers w/ 4 years of Mathematics
  - NATION: 529 on mathematics portion
  - GEORGIA: 500 on mathematics portion
- What kind of Mathematics are they taking?

Georgia's Student Achievement in Mathematics

Course Work	NATION	GEORGIA
Algebra	517	495
Geometry	519	498
Trigonometry	553	520
Precalculus	571	557
Other Mathematics Courses	510	487
Computer Mathematics	539	479
Calculus	608	584
AP/Honors Courses	599	585

International Student Achievement in Mathematics

Program for International Student Assessment (Mathematics)

- **TOP 5 SCORES**
  - Finland: 544
  - South Korea: 542
  - Netherlands: 538
  - Japan: 534
  - Canada: 533
  - INTERNATIONAL AVERAGE: 500
- **UNITED STATES**
  - Average Score: 483
  - 24th out of 30 countries
  - Well below International Average

Mandate from Georgia State Board of Education

Develop a curriculum that is rigorous, deep, provides clear expectations for students, is an instructional guide for teachers, and is student-focused rather than teacher-focused.

The Process

- Expert Advisory Panel
  - 15 leaders in education, government, business, and industry
  - studied curricula from high achieving states and nations
  - chose Japanese curriculum for leanness, rigor, and coherence

The Process

- Teacher writing teams
  - Teacher applications
  - Five days of training in use of performance standards
  - Began writing in July of 2003
- K-12 curriculum posted for 60 days for public review and comment in March of 2004.
- K-8 curriculum passed by SBOE in May of 2004; 9-12 adoption postponed.

The Process

- High School Advisory Committee
  - Formed in summer, 2004
  - 35 individuals - teachers, state and national leaders in mathematics education, higher education faculty
  - Revised drafts of 6-12 curriculum
  - Committee statement along with names and responsibilities of committee members posted at www.gadoe.org
- Posted for public review and comment for 60 days

The Process

- Reviewed by Dr. Ann Shannon, Research Scientist, Shell Centre, Nottingham, England
- Reviewed by the Board of Regents' Academic Advisory Committee for Mathematical Subjects
- Revised based on all reviews
- Adopted by unanimous vote of the Georgia State Board of Education, May 2005

Endorsements

- Board of Georgia Council of Teachers of Mathematics
- Senior Vice Chancellor for Academic Affairs of the Board of Regents
- Board of Regents' Academic Advisory Committee on Mathematical Subjects

Partnerships & Collaboration

- **University System of Georgia**
- **Georgia Department of Technical and Adult Education**
- **Governor's Office of Workforce Development**
- **Regional Education Services Agencies**
- **Georgia Council of Teachers of Mathematics**
- **Georgia Public Broadcasting**

Alignment with National Organizations

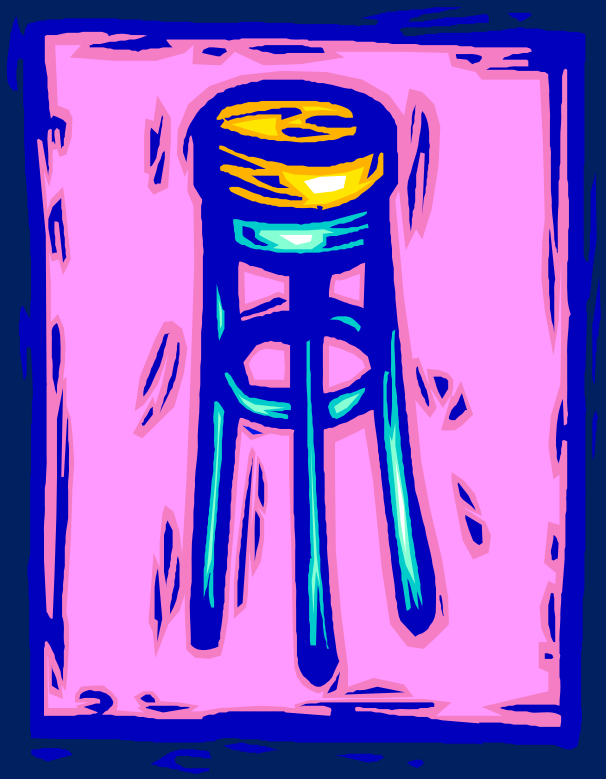
- National Council of Teachers of Mathematics
- College Board
- American Statistical Association
- Achieve

Implications for the Classroom

Students will be able to…

- Actively engage in mathematics
- Explain their thinking
- Justify their work
- Use multiple representations
- Make connections
- Choose appropriate technology

What Is It?



Balance of concepts, skills, and problem solving

A balance...

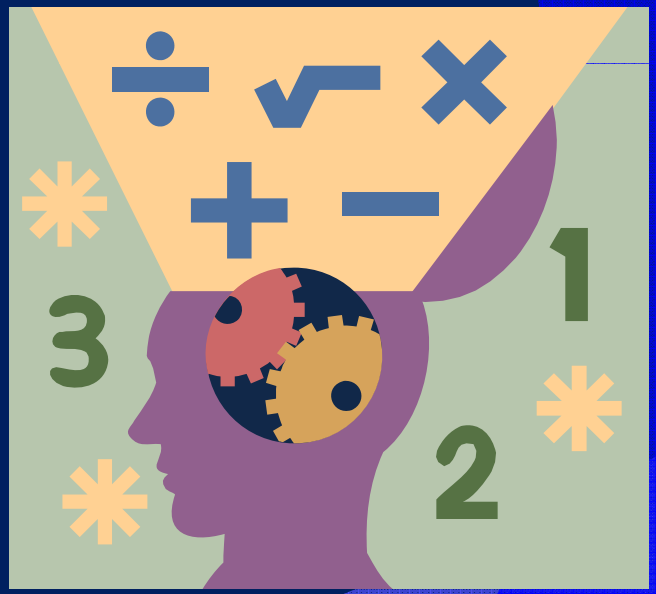
emphasizing
understanding and relevance

K-8 Mathematics

- K-2: Four strands: number and operations, measurement, geometry, and data analysis
- 3-5: Algebra strand is added
- 6-8: In-depth treatment of algebra and geometry begins in grade 6; 80% of concepts taught in traditional algebra 1 course and 60% of concepts taught in traditional geometry course completed by the end of grade 8

High School Mathematics

- Integrated curriculum
- Common level of mastery
- Multiple paths of study



How is it different?

Comparison of the Sheer Number of Standards Expected to Be Learned in a Year

Grades	Old (QCC)	New (GPS)
6th Grade	**53**	**18**
7th Grade	**43**	**15**
8th Grade	**45**	**18**

High School Course Chart

Mathematics	Accelerated Mathematics
Mathematics I	Accelerated Mathematics I
Mathematics II	
Mathematics III	Accelerated Mathematics II
Mathematics IV	Accelerated Mathematics III
	Other courses available: Discrete Mathematics Advanced Placement Statistics Advanced Placement Calculus AB Advanced Placement Calculus BC

A Ladder not a Spiral

Climbing the Ladder

- 7th-Cross sections and shadows
- 8th-Surface area of pyramids and cones as an application of the Pythagorean Theorem
- Mathematics 1-Comparing quadratic and cubic functions using surface area and volume of prisms, pyramids, cylinders, and cones
- Mathematics 2-Comparing quadratic and cubic functions using surface area and volume of spheres

A Ladder not a Spiral

Climbing the Ladder

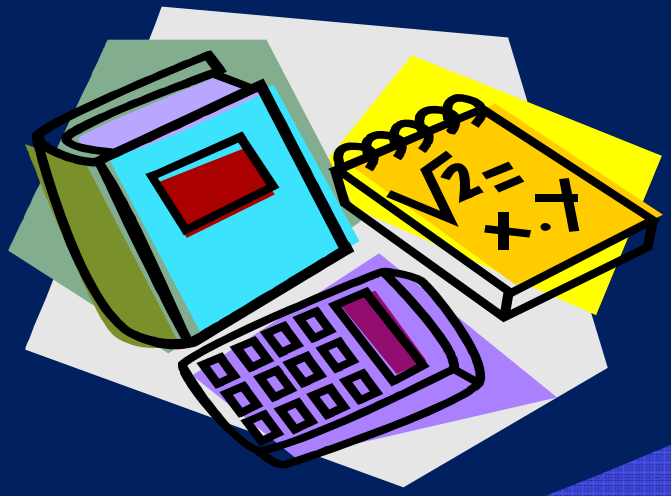
- 7th –Define and operate with absolute value
- 8th—Solve absolute value equations and inequalities
- Mathematics 1—Examine the absolute value function
- Mathematics 2— Explore the absolute value as a piecewise function

Performance Standards

- Content and process standards
- Tasks
- Student work
- Commentary

Content Standards

- Body of knowledge
- What students should know and be able to do



Process Standards

Promote mathematics literacy through:

- Problem solving
- Reasoning and proof
- Communication
- Connections
- Representations

Tasks

- Give detail to the elements of the content standards
- Provide depth of understanding
- Maintain high cognitive demand
- Define academic rigor of standards
- Exemplify the kind of performance expected of students
- Address group worthiness

Student Work

- Reflects the level a student should attain by the end of a grade or course
- Further defines the content standards
- Illustrates the kind of performance expected of students
- Relates to a strand or topic rather than a single standard, embodying many concepts

Commentary

- Identifies the mathematics involved in the task
- Identifies evidence of understanding related to a specific standard
- Informs the teacher in understanding the depth, detail and rigor expected in work that meets the standard
- Guides students in comparing and judging the quality of their own work

WHEN does it all happen?

Year	Teacher Training	Classroom Implementation
2004	Grade 6	
2005	Grades K-2, 7	Grade 6
2006	Grades 3-5, 8	Grades K-2, 7
2007	Math I	Grades 3-5, 8
2008	Math II	Grade 9
2009	Math III	Grade 10
2010	Math IV	Grade 11
2011		Grade 12

Start with a belief

Teachers must believe that all students can learn, although in different ways and at different rates!

Instructional Elements

- A meaningful mathematics curriculum
- An emphasis on interactive endeavors that promote divergent thinking within a classroom
- Diversified instructional strategies that address the needs of all types of learners

- Assessment that is varied, ongoing, and embedded in instruction
- Focused lesson planning that, instead of emphasizing what the classroom teacher wants to teach, begins by understanding what students need to learn and assessing what they already know

EDThoughts: What we Know About Mathematics Teaching and Learning

Which item requires a better understanding of lines?

1. Given a slope of 5 and a y-intercept of 3, write the equation of the line.

OR

2. A company that produces pens has n pens in stock at the beginning of a certain day. It produces these pens at a constant rate r per hour for the entire day. If that day, pens have been produced at a greater constant rate, write an equation that can be used to determine the number of pens the company has in stock at the end of that day.

Mathematics Standard Example

MA1A1. Students will explore and interpret the characteristics of functions, using graphs, tables, and simple algebraic techniques.

a. Represent functions using function notation.

How to Read the GPS Code

MA1A1a.

Mathematics A**a**ccelerated
Mathematics I Algebra
Standard #**1**
Element **a**

Standards and Elements

- **Standard is in bold print**: Sets the parameters.
- **Elements are listed under the standard:** Set the expectations for understanding and communicates what the student should **know and be able to do**.

Mathematics I

- Family of Functions
  - Characteristics of the functions

    $F(x) = x^n$ (n=1,2,3), $\sqrt{x}$, $|x|$, and $1/x$
  - Sequences as functions

Mathematics I

- Algebra of Quadratics
  - Factoring of 2nd degree polynomials & cubes
  - Quadratic equations
  - Radical equations
  - Simple rational equations
- Coordinate Geometry
  - Distance between a point and a line
  - Midpoint

Mathematics I

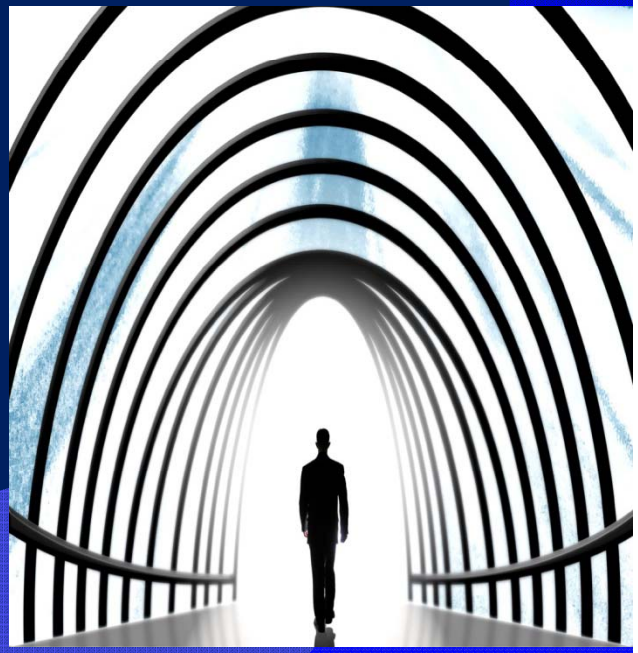
- Triangles
  - Inductive, deductive reasoning
  - Converse, inverse, contrapositive
  - Sum of interior, exterior angles
  - Triangle inequalities
  - SSS, SAS, ASA, AAS, HL
  - Incenter, orthocenter, circumcenter, centroid

Mathematics I

- Statistics
  - Simple permutations & combinations
  - Mutually exclusive and dependent events
  - Conditional probabilities
  - Expected values
  - Summary statistics
  - Random sample
  - Mean absolute deviation

Mathematics II

- Family of Functions
  - Quadratic ($y = ax^2 + bx + c$)
  - Step & piecewise
  - Exponential
  - Inverse
  - Characteristics of their graphs



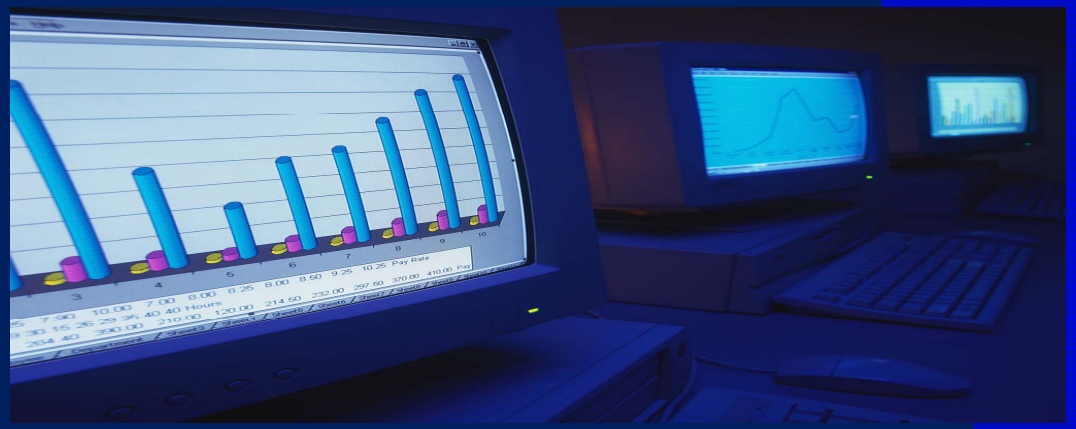
Mathematics II

- Complex numbers
- Quadratic inequalities
- Exponential equations and inequalities
- Geometric sequences as exponential functions
- Right triangle trigonometry
- Circles and properties

Mathematics II

- Length of arc
- Surface area and volume of sphere
- Relationships of similar solids
- Population means & deviations
- Modeling of data using linear and quadratic regressions

Mathematics III



- Circle
- Ellipse
- Hyperbola
- Parabola (concave right and left)
- Planes & spheres
- Histograms
- Normal distribution
- Experimental and observational studies

Mathematics III

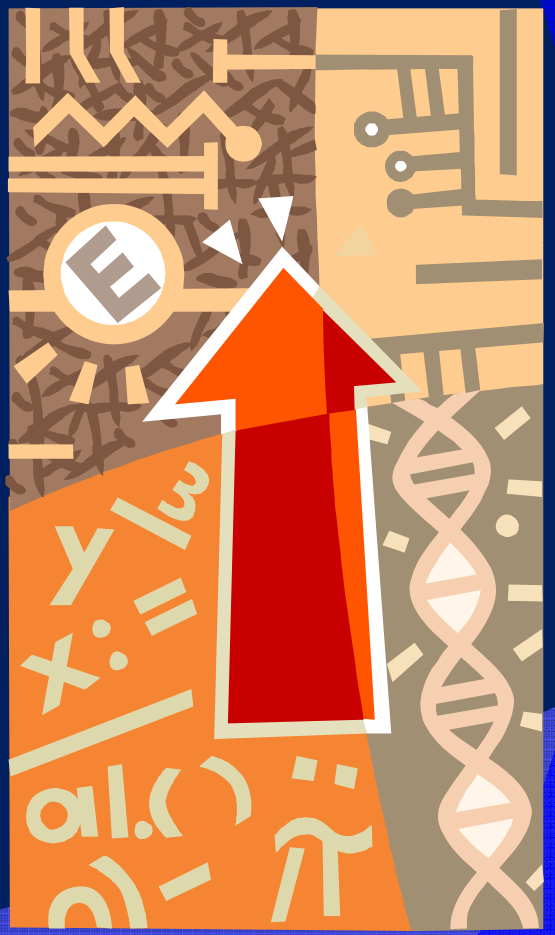
- Extension of exponents
- Matrices
- Polynomials of degree > 2
- Logarithmic functions
- Exponential, logarithmic and polynomial equations and inequalities
- Vertex-edge graphs
- Linear programming

Mathematics IV

- Vectors
- Graphs of 6 trigonometric functions
- Trigonometric identities
- Trigonometric equations and inequalities
- Rational functions
- Rational equations and inequalities
- Inverse trigonometric functions (sine, cosines, and tangent only)

Mathematics IV

- Sequences and series
- Unit circle
- Law of Sines
- Law of Cosines
- Area of triangle formula
- Central Limit Theorem
- Confidence interval
- Margin of error



The National Mathematics Advisory Panel Report

- P-8 Standards should be "streamlined" and "well-defined".

  "Any approach that revisits topics year after year without bringing them to closure should be avoided."

- A balance between concepts, computation and problem solving. They are "equally important and mutually reinforce each other."

- Proficiency with whole numbers, fractions and certain aspects of geometry and measurement are the foundations for algebra.

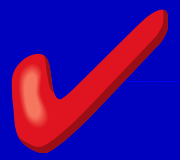
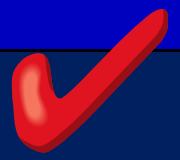
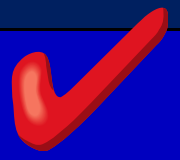
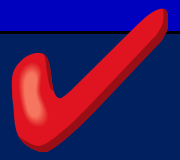
- More students should be prepared for and offered an authentic algebra course in Grade 8.

The National Mathematics Advisory Panel Report

Student Effort and High Expectations Matter!

"If children believe that their efforts to learn make them 'smarter,' they show greater persistence in mathematics learning."

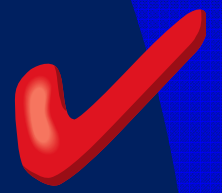
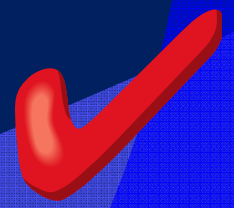
Recommended Benchmarks: Elementary School

By the end of Grade 3, students should be proficient with the addition and subtraction of whole numbers.	
By the end of Grade 4, students should be able to identify and represent fractions and decimals, and compare them on a number line or with other common representations of fractions and decimals.	
By the end of Grade 5, students should be proficient with multiplication and division of whole numbers.	
By the end of Grade 5, students should be proficient with comparing fractions and decimals and common percents, and with the addition and subtraction of fractions and decimals.	
By the end of Grade 5, students should be able to solve problems involving perimeter and area of triangles and all quadrilaterals having at least one pair of parallel sides (i.e.,	

Recommended Benchmarks: Middle Schools

By the end of Grade 6, students should be proficient with multiplication and division of fractions and decimals.	
By the end of Grade 6, students should be proficient with all operations involving positive and negative integers	Grade 7
By the end of Grade 6, students should be able to analyze the properties of two-dimensional shapes and solve problems involving perimeter and area, and analyze the properties of three-dimensional shapes and solve problems involving surface area and volume.	

Recommended Benchmarks: Middle Schools (continued)

By the end of Grade 7, students should be proficient with all operations involving positive and negative fractions.	
By the end of Grade 7, students should be able to solve problems involving percent, ratio, and rate and extend this work to proportionality.	
By the end of Grade 7, students should be familiar with the relationship between similar triangles and the concept of the slope of a line.	

The National Mathematics Advisory Panel Report

The Importance of Knowledgeable Teachers

- Preparation for Elementary and Middle School teachers in Mathematics should be strengthened
- “Teachers cannot be expected to teach what they do not know.”

Effective Instruction Matters

- Use of formative assessments
- The belief that children of certain ages are “too young” to learn math is false
- Use an array of examples in teaching and offer opportunities for extensive practice and the ability to “think aloud.”
- Accelerate gifted mathematics students

Morning Break

Constructing the Algebra Ladder

Climbing the Ladder

6th Grade Task

View Tube Experiment

7th Grade Task

Seesaw Nickels

8th Grade Task

Connecting Arithmetic Sequences and Linear Functions


Out to
LUNCH!

From Wonderland to Functionland

- **Be able to explain and describe what you should see happening during this lesson and task.**

- **Think about what needs to take place before students would be able to perform this task.**

Afternoon Break

From Wonderland to Functionland

- **Be able to explain and describe what you should see happening during this lesson and task.**

- **Think about what needs to take place before students would be able to perform this task.**

End of Day One