innovation in teaching competition beyond the textbook

THE GOVERNOR'S OFFICE of STUDENT ACHIEVEMENT

HANDS-ON APPLICATIONS OF MODELING GEOMETRY

BRIAN SWANAGAN, FLOYD COUNTY COLLEGE AND CAREER ACADEMY

Day One

Standards Addressed: M.9-12.A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.

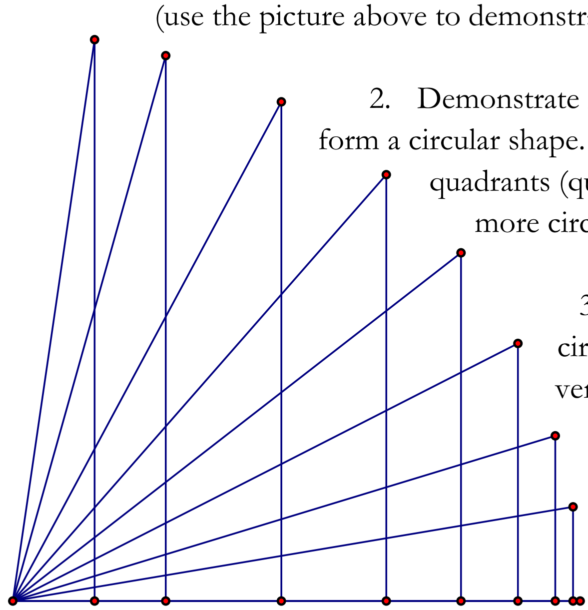
Review the concepts of linear and quadratic equations with students. The review should include comparing and contrasting linear and quadratic equations and solving systems of linear and quadratic equations using the quadratic formula. Have students practice these skills by hand.

Day Two

Standards Addressed: M.9-12.G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean theorem; complete the square to find the center and radius of a circle given by an equation.

Working in groups, students will make several paper triangles that have the same hypotenuse length. The triangles can be different sizes as long as the hypotenuse lengths are the same. As students work, be sure to address the following points with students:

1. Explain that the constant hypotenuse length represents the radius of a circle. Have the students glue their paper triangles together so the acute angles meet at the center of the circle (use the picture above to demonstrate to students how to align the triangles).



2. Demonstrate how the vertices of the other acute angles in the triangles form a circular shape. Encourage students to place their triangles in a variety of quadrants (quadrants I, II, III, and IV) so that the vertices form a more circular shape.

3. Discuss how the coordinates of the points on the circle are defined by a unique horizontal distance (x) and vertical distance (y), but the radius (hypotenuse of the right triangle) associated with each point is constant.

4. Explain how the Pythagorean theorem applies to the triangles the students created because all of the triangles are right triangles.

5. Use the Pythagorean theorem to help students derive the equation for a circle by recognizing that for each point on the circle, x, y, and r have the following relationship: $x^2 + y^2 = r^2$.

6. Have students apply the distance formula and their prior knowledge of function shifts to explain the effects of vertical and horizontal shifts on the equation of a circle.

Day Three*

Standards Addressed: M.9-12.G.GPE.2 Derive the equation of a parabola given a focus and a directrix.

> **This lesson assumes that the students have a 90 minute block schedule. If the class is only fifty minutes long, stop after part two and begin part three the next class.*

Part One: (5 to 10 minutes)
Students complete a short warm-up exercise at the start of class. This warm-up can either be review or preparation for today's lesson.

Part Two: (30 minutes)
Take the students outside or to a large, clear space indoors. Make sure to bring at least forty feet of rope (or yarn or large paper) and two tape measures. Give instructions to help students form a parabola by positioning themselves correctly. Students will be forming four different parabolas. Each parabola is outlined below.

<u>First Parabola</u>

1. Have two students grab the ends of the rope. Then, straighten the rope so that it forms a line. Tell students that the rope represents the *directrix*.

2. Have one student stand ten feet away from the center of the rope on one side (the student can choose the side). Tell the students that this student represents the *focus*.

3. Designate two students as the "measurers" who will be using the measuring tapes to help students determine how far they are away from the directrix and the focus. Instruct the remaining students to attempt to stand in positions that are equidistant from the directrix and the focus. Students will work with the measurers to verify their distances and adjust their positions until the distances are equal. Make sure students are measuring their perpendicular distance to the directrix when they are determining where to stand. If necessary, discuss with students the challenges of measuring distance to a line and the reason for using the perpendicular distance. Guide students to choose positions so that there are a balanced number of students on each side of the focus. Together, the students should form a parabola.

4. Once the students have found their positions, discuss the following concepts with the students: *symmetry, vertex, distance to a line, equidistance*, and the *shape of a parabola*. Incorporate the students' positions into the discussion to help students visualize the concepts associated with a parabola.

Second Parabola

1. Have the students hypothesize what will happen if the focus moves left or right while remaining parallel to the directrix. Ask them to explain their hypotheses using mathematical reasoning.

2. Ask the student representing the focus to move about four feet to the right or left while remaining parallel to the directrix. Instruct the other students to modify their positions so that they are equidistant from the focus and directrix as they did with the first parabola. Have the measurers help students verify their positions and adjust where they are standing if necessary.

3. Ask the students in the parabola how they know they are still equidistant to the focus and directrix as they move parallel to the directrix in any direction. Discuss with students how they adjusted their own positions in response to the movements of the focus. Have the students reference their physical movements to help them visualize the shifting parabola.

Third Parabola

1. Have the students hypothesize what will happen if the focus moves away from the directrix in a perpendicular direction. Ask them to explain their hypotheses using mathematical reasoning.

2. Ask the student representing the focus to move about five feet away from the directrix in a perpendicular direction. Instruct the other students in the parabola to modify their positions so that they are equidistant from the focus and directrix. Have the measurers help students verify their positions and adjust where they are standing if necessary.

3. Discuss with students the general shape of a parabola, the transformations that have occurred since the first parabola, and how the vertex has moved. Ask the students the following questions:
    - *What would happen if we moved the focus closer to the directrix?*
    - *Why are the effects of perpendicular movements different than the effects of parallel movements?*
    - *What would need to happen so that everyone could simply shift vertically up and down together?*
    - *If the focus and directrix need to move up and down together, why did this not seem to happen when we moved the parabola left and right?**

**When discussing question four, tell the students that the directrix is a line that goes on forever, so whether it does or does not move to the right or left as the focus moves, it will not be noticeable.*

<u>Fourth Parabola</u>

1. Have the students hypothesize what will happen if the directrix changes direction or if the focus moves to the other side of the directrix. Ask them to explain their hypotheses using mathematical reasoning.

2. Ask one of the students holding the directrix to move so that the angle changes. Ask everyone in the parabola to move their position in response so that they are still equidistant from the focus and directrix. Have the measurers help students verify their positions and adjust where they are standing if necessary.

3. Discuss with students how they adjusted their own positions in response to the movements of the focus. Have the students reference their physical movements to help them visualize the changing parabola.

4. Next, ask the student representing the focus to move to the opposite side of the directrix (or ask both students holding the rope to move to the other side of the focus). Ask everyone in the parabola to move their position in response so they are still equidistant from the focus and directrix. Have the measurers help students verify their positions and adjust where they are standing if necessary. Discuss with students how they adjusted their own positions in response to the movements of the focus. Have the students reference their physical movements to help them visualize the changing parabola.

5. Have a discussion with the students about the vocabulary terms directrix and focus. Talk about how each term may have gotten its name in relation to how it affects the shape and direction of the parabola.

Part Three: (10 minutes)

1. Instruct students to head over to the 3D object shaped like a satellite dish. The object is a 3D parabolic solar reflector. You can construct the parabolic solar reflector on your own, or you can make one using a satellite dish covered with extremely reflective material. If your school has a metals instructor, ask them to build one for the class with the specified measurements.

2. Tell the students the solar reflector represents a paraboloid. Demonstrate the power of a parabolic solar reflector* by placing various objects at the focus of the parabolic reflector and heating them up. Objects can include: paper, water, wood, marshmallows, hotdogs, and

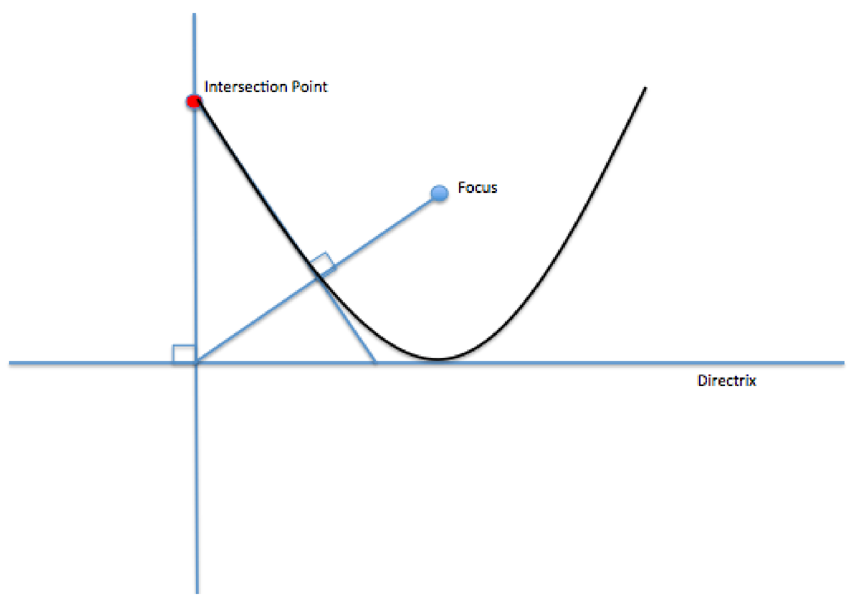
much more. Make sure the objects chosen are safe to heat up. Work with science faculty at the school as needed to determine which objects would be good exhibits. Make sure are students are safe throughout the presentation.

> **There are other demonstrations that can showcase the capabilities of a paraboloid. One alternative is to put a microphone where the focus is and cover the entire reflector with sound-reflective material such as vinyl. Students will be able to listen to various sounds at a distance. Another alternative is to put a light where the focal point is to create a giant flashlight. This alternative is especially well suited for cloudy weather.*

3. During the demonstration, discuss the importance of the paraboloid's focus in the heating process. Relate this discussion to the parabolas the students formed before by standing in different positions.

Part Four: (10 to 15 minutes)

1. Return to the classroom and lead a Geometer SketchPad (GSP) or GeoGebra demonstration for the students on what they just experienced outside. The demonstration can be projected on a screen in the classroom.



2. Construct a horizontal line that will serve as the directrix. Remind students that this was the rope in the outdoors activity. Construct a point (focus) somewhere above the center of the directrix. Remind students that in order to form a parabola, each student had to pick a point that was equidistant from the focus and the directrix. Pick a point on the line that is to the left or right of the focus and construct a perpendicular line through that point. Explain that this perpendicular line represents one of the measuring tapes used outside. Tell the students that the goal is to find some point on the perpendicular line that is the same distance from the directrix and the focus.

3. Explain that geometry can help determine where this point lies. Construct a line segment between the focus and the point on the directrix that the perpendicular line is going through. Draw a perpendicular bisector to this segment. Any point on this bisector is equidistant between the endpoints of the segment. Thus, the intersection of this perpendicular bisector

and the perpendicular line to the directrix is a point equidistant between the directrix and the focus. This intersection point represents one point on the parabola associated with the given directrix and focus.

4. To prove that the intersection point is a point on the parabola, draw an additional segment from the intersection point to the focus. This action creates two triangles that can be proven congruent using the SAS congruency theorem due to the shared side in between, the right angles, and the equal bases created by the perpendicular bisector. This means that the hypotenuses of the two triangles are congruent. Since the hypotenuses represent both the distance from the intersection point to the focus and the intersection point to the directrix, the triangles prove that the intersection point is equidistant between the focus and directrix.

5. Next, trace the intersection point while moving the line perpendicular to the directrix from right to left. Tracing should reveal a parabolic shape. While examining the traced parabola, explain that the perpendicular to the directrix can represent the light from the sun entering the parabolic reflector from above. The angle formed by the perpendicular and the parabola is congruent to the angle formed by the parabola and the segment from the parabola to the focus using the transitive property. Tell students that this means all of the rays of light coming in perpendicular to the directrix will reflect directly to the focus along these angles. Explain that this is the reason why objects could be heated using the parabolic reflector by placing them where the focus is. Make sure to emphasize that the incoming rays of light must be perpendicular.

6. Next, remove the traces and construct a locus using the intersection point and the point on the directrix. The locus will clearly display the parabola associated with the given focus and directrix. Move the focus from left to right to demonstrate horizontal shifts of parabolas. Ask students to relate this movement to what students observed during the outdoors activity in Part Two. Next, move the focus closer to or away from the directrix to demonstrate vertical stretches and shrinks of parabolas. Ask students to relate this movement to what students observed during the outdoors activity in Part Two. Move the directrix so it is at different angles to demonstrate rotations of parabolas. Again, ask students to relate this movement to what students observed during the outdoors activity. Finally, move the focus so that it is on the other side of the directrix to demonstrate a reflection. Ask students to relate this movement to what students observed outdoors. Discuss these patterns with students and have them make connections between the vertex, focus, and directrix of a parabola and the effects of moving the focus and/or directrix on a parabola.

Part Five: (10 minutes)

Students complete a brief activity using the **Parabolas Day One Activity Sheet and Homework, #1**) where they fill in one of the three missing pieces (focus, vertex, and directrix) given the other two. Have students present their results on the classroom board and discuss any noteworthy misconceptions.

Part Six: (remainder of class time)

Model for students how to fill in one of the three missing pieces (focus, vertex, directrix) on a coordinate grid (**Parabolas Day One Activity Sheet and Homework, #2**). Have students complete this activity at home.

Optional: Have students complete the rest of the activity that helps them derive the equation of a parabola using a generic point and the distance formula (**Parabolas Day One Activity Sheet and Homework, #3-5**). This assignment can be a differentiation opportunity by having accelerated students complete the activity independently. Otherwise, this portion of the activity can be completed at the beginning of the next lesson.

Day Four

Standard Addressed: M.9-12.G.GPE.2 Derive the equation of a parabola given a focus and a directrix.

Part One: (5 to 10 minutes)

Have the students complete a warm-up that requires them to determine the coordinates or equation of a missing piece (either focus, vertex, or directrix) given the other two.

Part Two: (10 to 15 minutes)

Review the homework (**Parabolas Day One Activity Sheet and Homework**). If you did not assign numbers three through five previously, derive the generic equation of a parabola together as a class. Ask the class to explain why the equation for a parabola will always be a quadratic equation.

Part Three (20 to 30 minutes)

1. Introduce the gallery walk assignment (**Parabolas Day Two Station Questions)**. Tell students there will be four stations around the room. At each station, students will observe graphs of parabolas in which the focus and/or directrix have been changed in similar ways (i.e. shifted vertically, shifted horizontally, etc.).

2. Have students work in small groups to discover patterns and relationships among the focus, vertex, directrix, parabola, and equation for the parabola. Students should record their answers to the **Parabolas Day Two Station Questions.**

3. Have the class come together to discuss their findings. Use this discussion to collectively establish a generic formula that relates the focus, vertex, and directrix of a parabola. This will be the generic equation for a parabola using the focus, vertex, and directrix.

Part Four: (10 - 15 minutes)

Students will build their own basic parabolic reflectors that they will use to cook hotdogs. In order to do so, students first need to derive the parabolic equation they will use to build the hotdog cooker.

1. Give the students constraints that will help define the location of the directrix and focus for the paraboloid. Have students use the generic formula for a parabola to derive a formula for the parabolic reflector with the given constraints.

2. Give students cardboard boxes to construct their hotdog cookers. Instruct them to use the dimensions of the box and the given constraints to determine where the focus of their parabola will be. Suggest that the students define the origin of their coordinate system to be located at the vertex and parallel to the directrix.

3. If a metals department in the school built the parabolic reflector for the demonstration on Day Three, provide students with the measurements the metals department used as an example. For instance, tell students that the metals department was told to place the focus two feet above the vertex and make the parabola four feet wide. Given these constraints, ask the students how to determine the height of the parabola at each end. Instruct students to use similar logic in determining the equations and measurements for their own parabolic reflectors.

Part Five: (20 to 30 minutes)

Allow students to build their own parabolic solar reflectors using cardboard, aluminum foil, wooden skewers, tape, and straws. The straws can help determine the angle of the solar reflector to the sun. Depending on the amount of class time available, this activity may continue into another lesson day. Students can also finish building the paraboloids at home. When the parabolic reflectors are completed, have students cook hotdogs with their solar reflectors to conclude the lesson.

For further directions on how to construct hot dog cookers, please visit: http://www.education.com/science-fair/article/solar-hot-dog-cooker/