Name: $\qquad$

## The Cost of College Education Project Packet

## Introduction

We live in a society where a college education is considered the norm and not the exception. While everyone is expected to attend a college or university (of their choice), most American families don't have a plan for paying for this expectation. According to Sallie Mae's "How America Saves for College 2013" study, just 36 percent of middle-income families and 29 percent of low-income families have set aside money to pay for furthering their education.

As Grandma, Helen, watched her proud son hold her beautiful new granddaughter Jessica, she thought, "What can I do help her live the wonderful life she so richly deserves?" The grandmother reflected on the times she and her husband struggled to keep their son in school and what a financial burden the student loans had become for her son. Because her son saw no way out of this financial maze the opportunity for a family seemed almost nonexistent. She realized that there was a real possibility she could have never experienced this type of joy. She quickly realized that this was not the fate she wanted her granddaughter to suffer. She knew she had to do something to not have her granddaughter suffer the same fate.

## Mathematical Goals

- Represent data on a scatter plot
- Describe how two variables are related
- Use technology to graph and analyze functions
- Informally assess the fit of a function by plotting and analyzing residuals


## Essential Question

How can students predict and prepare for the cost of a college education?
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## Part 1: Future Value

The Grandmother, Helen, is looking for a plan to finance her new grandchild Jessica's college education. She is planning to invest $\$ 50,000$ on Jessica's first birthday July 1, 2008. Your role is to assist the Grandma to find an investment plan which has to offer compound interest for the investment. In addition, you need to predict Jessica's future college education cost by using the available data.

Use the internet or any reliable sources, find the interest rate and the number of compounding periods per year advertised by a bank.

- The principal ( $\boldsymbol{P}$ ) is $\$ 50,000$.
- Research the annual interest rate $(\boldsymbol{r})$ for the investment. $\mathrm{r}=$ $\qquad$
- State the time ( $\boldsymbol{t}$ ) in years for the investment (as in when Jessica will be attending her first year of college). $\mathrm{t}=$ $\qquad$
- State the number of compounding periods ( $\boldsymbol{n}$ ) per year. $\mathrm{n}=$ $\qquad$

1. Write a mathematical model for the future value of Grandma's investment as an exponential function, with time ( t ) as the independent variable. $\mathrm{Use} F(t)=P(1+r / n)^{n t}$
2. Write the growth factor for this investment. Round your answer up to five decimal places. ( Note: growth factor $=1+r / n$ )
3. Calculate the future value of Grandma's investment on Jessica's $17^{\text {th }}$ birthday. Round your answer to the nearest dollar.
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## Part 2: College Tuition and Fee Trend Table

FIGURE 7 Published Tuition and Fees, Net Tuition and Fees, and Room and Board in Constant 2010 Dollars, Full-Time Undergraduate Students, 1995-96, 2000-01, 2005-06, and 2010-11 (Estimated)


The distribution of grant aid across sectors and between full-time and part-time students is based on data from the National Postsecondary Student Aid Study. The distribution for 2009-10 includes estimates of the changes resulting from the large increase in Pell Grants that year. Because financial aid data for 2010-11 are not yet available, net prices for 2010-11 are estimated based on past years and available information about changes in financial aid.

NOTE: Numbers have been rounded to the nearest $\$ 10$.
SOURCES:The College Board, Annual Survey of Colleges; Trends in Student Aid 2010; calculations by the authors.
For detailed background data and additional information, please visit http://trends.collegeboard.org.
4. After celebrating her $17^{\text {th }}$ birthday, Jessica is planning to attend a private nonprofit four year college with room and board during the fall semester. Visit the following website and complete the table starting from the 2006-2007 academic year.
http://trends.collegeboard.org/college-pricing/figures-tables/tuition-and-fee-and-room-and-board-charges-over-time (Students may refer to Page 13 to answer this question)

| Academic <br> Year | Years(x) | Average cost per <br> year(\$) <br> (in 2013 Dollars $)$ |
| :--- | :--- | :--- |
| $2006-2007$ | 0 | 35007 |
| $2007-2008$ | 1 |  |
| $2008-2009$ | 2 |  |
| $2009-2010$ | 3 |  |
| $2010-2011$ | 4 |  |
| $2011-2012$ | 5 |  |

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| $2012-2013$ | 6 |  |
| :--- | :--- | :--- |
| $2013-2014$ | 7 |  |

5. Draw the scatter plot for the above data by taking the time $(\mathrm{x})$ in years on the x - axis and the average cost per year (in thousands) on the y-axis.

6. Which BEST describes the correlation of the two variables shown in the scatter plot above?
A. weak positive
B. strong positive
C. weak negative
D. strong negative
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## Part 3: Linear Regression Model

7. Use a technological tool to find the linear regression model for No. 4. Take the number of years (x), starting from academic year 2006-2007, as an independent variable and the corresponding average cost per year as the dependent variable. Write your answer in the form $\mathrm{f}(\mathrm{x})=\boldsymbol{a} \mathrm{x}+\boldsymbol{b}$. Round the values of $\boldsymbol{a}$ and $\boldsymbol{b}$ to the nearest whole numbers. Interpret the values of $\mathbf{a}$ and $\boldsymbol{b}$ in the context of this problem.
8. Draw the graph of the above linear regression model on the grid provided for No. 5 .
9. In which academic year will Jessica be attending her first year of college?
*Use the linear regression model from No. 7 to answer questions 10-13. For No. 13-17, round your answer to the nearest dollar.*
10. Predict Jessica's average cost for her first year's college education.
11. Predict Jessica's average cost for her second year.
12. Predict Jessica's average cost for her third year.
13. Predict Jessica's average cost for her fourth year.
14. What would be the total cost for all four of her college years?

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15. Jessica's institution is offering a $10 \%$ discount on the total fee if a student pays the fee for all four years in advance at the beginning of the first year. If she decides to pay the full amount in advance, what amount would she owe?
16. Does grandma's investment cover Jessica's total cost of her four year college education? Explain.
17. If not, how much more money should her grandma have invested on Jessica's first birthday?
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## Part 4: Exponential Regression Model

18. Use a technological tool to find the linear regression model for No. 4. Take the number of years (x), starting from academic year 2006-2007, as an independent variable and the corresponding average cost per year as the dependent variable. Write your answer in the form $\mathrm{g}(\mathrm{t})=a * b^{x}$. Round the value of $\boldsymbol{a}$ to the nearest whole number and $\mathbf{b}$ to the nearest six decimal places. Interpret the values $\mathbf{a}$ and $\mathbf{b}$ in the context of this problem.
*Use the above exponential regression model from No. 18 to answer questions19-22.
For No. 19-26, round your answers to the nearest dollar.*
19. Predict Jessica's average cost for her first year's college education.
20. Predict Jessica's average cost for her second year.
21. Predict Jessica's average cost for her third year.
22. Predict Jessica's average cost for her fourth year of college.
23. What would be the total cost for all of her four years in college?
24. As you know, Jessica's institution is offering a $10 \%$ discount on the total fee if a student pays all four years fees at the beginning of the first year. Jessica and her grandma decided to opt for this discount program. How much Jessica will end up paying after discount?
25. Do you think, Grandma's investment will provide sufficient funds to pay Jessica's college cost/fee as you calculated in No.24? Justify.

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26. If not, how much more money should her Grandma have invested on Jessica's first birthday?

## Part 5: Interpretation and Summarization

The goodness of fit depends on the model's accuracy in predicting values. Residuals, or error distances, are used to measure the goodness of fit. A residual is the difference between the observed value and the model's predicted value. For a regression model, a residual = observed value predicted value. A residual plot is a graph that shows the residual values ( $y$ ) on the vertical axis and the independent variable $(x)$ on the horizontal axis. A residual plot shows where the model fits best, and where the fit is worst. A good regression fit has very short residuals.
27. Calculate the residuals for the observed cost of education and the predicted cost of education by using the linear regression model you obtained from No. 7.

| Academic <br> Year | Years(x) | Average cost <br> per year(\$) <br> (Observed cost) | Predicted cost <br> per year(\$) | Residual |
| :---: | :---: | :---: | :---: | :---: |
| $2006-2007$ | 0 | 35007 |  |  |
| $2007-2008$ | 1 |  |  |  |
| $2008-2009$ | 2 |  |  |  |
| $2009-2010$ | 3 |  |  |  |
| $2010-2011$ | 4 |  |  |  |
| $2011-2012$ | 5 |  |  |  |
| $2012-2013$ | 6 |  |  |  |
| $2013-2014$ | 7 |  |  |  |

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28. Construct a residual plot for the above data by taking years on the x -axis and the corresponding residuals (in bundreds) on the y -axis.

29. Determine if there are any patterns in the above residual plot and explain what they suggest about the relationship between the variables.
30. What is the sum of the residuals? Interpret the result.
31. Do you think that the linear regression model you obtained is a good predictor of cost of education? Why or why not?

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32. Use a technology tool to calculate the correlation coefficient for the two variables, number of years ( $t$ ) and the average cost of education. What can you conclude?
33. Calculate the residuals for the observed cost of education and the predicted cost of education by using the exponential model you obtained from question 18.

| Academic <br> Year | Years(x) | Average cost <br> per year(\$) <br> (Observed <br> value) | Predicted cost per <br> year(\$) | Residual |
| :---: | :---: | :---: | :---: | :---: |
| $2006-2007$ | 0 | 35007 |  |  |
| $2007-2008$ | 1 |  |  |  |
| $2008-2009$ | 2 |  |  |  |
| $2009-2010$ | 3 |  |  |  |
| $2010-2011$ | 4 |  |  |  |
| $2011-2012$ | 5 |  |  |  |
| $2012-2013$ | 6 |  |  |  |
| $2013-2014$ | 7 |  |  |  |

34. Construct a residual plot for the above data by taking years on the x -axis and the corresponding residuals (in bundreds) on the y -axis.

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35. Determine if there are any patterns in the above residual plot and explain what they suggest about the relationship between the variables.
36. What is the sum of the residuals obtained using the exponential model? Interpret your answer.

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37. Do you think the exponential regression model you obtained is a good predictor of cost of education? Why or why not?
38. Which one of these two models (linear or exponential) do you prefer? Justify your preference.
39. Using a graphing utility, draw the scatter plot for question No. 4, graph both linear and exponential models in a same grid. What is your observation?
40. Write a paragraph about how this project will help you to plan for your college education.
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Trends in Higher Education

## Home $\quad$ Trends in College Pricing $\quad$ Trends in Student Aid $\quad$ Education Pays

Trends in College Pricing > Figures \& Tables > Published Prices - National > Tuition and Fee and Room and Board Charges over Time
Tuition and Fee and Room and Board Charges over Time

Other Data in this Topic
Online Only!
The annual increase in inflation-adjusted average tuition and fees at public four-year colleges and universities has declined in each of the past five years, from $9.5 \%$ in 2009-10 to 0.9\% in 2013-14.

| Academic Year | Tuition and Fees in 2013 Dollars |  |  | Tuition, Fees, Room and Board in 2013 Dollars |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Private Nonprofit Four-Year | Public Four-Year | Public Two-Year | Private Nonprofit Four-Year | Public Four-Year |
| 71-72 | \$10,515 | \$2,456 | \$1,102 | \$16,811 | \$8,064 |
| 81-82 | \$10,489 | \$2,318 | \$1,107 | \$16,143 | \$7,319 |
| 91-92 | \$16,829 | \$3,614 | \$2,008 | \$24,334 | \$9,351 |
| 01-02 | \$22,869 | \$4,956 | \$2,116 | \$31,395 | \$11,886 |
| 06-07 | \$25,607 | \$6,662 | \$2,601 | \$35,007 | \$14,735 |
| 07-08 | \$26,264 | \$6,943 | \$2,573 | \$35,878 | \$15,204 |
| 08-09 | \$26,356 | \$7,008 | \$2,530 | \$35,895 | \$15,263 |
| 09-10 | \$27,920 | \$7,672 | \$2,787 | \$38,041 | \$16,526 |
| 10-11 | \$28,679 | \$8,174 | \$2,938 | \$39,072 | \$17,335 |
| 11-12 | \$28,830 | \$8,557 | \$3,074 | \$39,261 | \$17,739 |
| 12-13 | \$29,557 | \$8,816 | \$3,216 | \$40,220 | \$18,166 |
| 13-14 | \$30,094 | \$8,893 | \$3,264 | \$40,917 | \$18,391 |


| Academic Year | Tuition and Fees in Current Dollars |  |  | Tuition, Fees, Room and Board in Current Dollars |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Private Nonprofit Four-Year | Public Four-Year | Public Two-Year | Private Nonprofit Four-Year | Public Four-Year |
| 71-72 | \$1,832 | \$428 | \$192 | \$2,929 | \$1,405 |
| 81-82 | \$4,113 | \$909 | \$434 | \$6,330 | \$2,870 |
| 91-92 | \$9,812 | \$2,107 | \$1,171 | \$14,188 | \$5,452 |
| 01-02 | \$17,377 | \$3,766 | \$1,608 | \$23,856 | \$9,032 |
| 06-07 | \$22,308 | \$5,804 | \$2,266 | \$30,497 | \$12,837 |
| 07-08 | \$23,420 | \$6,191 | \$2,294 | \$31,993 | \$13,558 |
| 08-09 | \$24,818 | \$6,599 | \$2,382 | \$33,800 | \$14,372 |
| 09-10 | \$25,739 | \$7,073 | \$2,569 | \$35,070 | \$15,235 |
| 10-11 | \$26,766 | \$7,629 | \$2,742 | \$36,465 | \$16,178 |
| 11-12 | \$27,883 | \$8,276 | \$2,973 | \$37,971 | \$17,156 |
| 12-13 | \$28,989 | \$8,646 | \$3,154 | \$39,447 | \$17,817 |
| 13-14 | \$30,094 | \$8,893 | \$3,264 | \$40,917 | \$18,391 |

