Rules of Exponents

An **exponent** is something that raises a number or a variable to a power. It is a process of repeated multiplication. For example, the expression 2^3 means to multiply 2 times itself 3 times or $2 \bullet 2 \bullet 2 = 8$. In 2^3, the 2 is called the **base** and the 3 is called the **exponent**. Both the base and the exponent can be either a number or a variable.

Each of the following is an example of an **exponential expression**.

3^4
x^5
4^t
k^3
2^{x-3}

When both the base and the exponent are numbers, we can evaluate the expression as we did with $2^3 = 2 \bullet 2 \bullet 2 = 8$. If either the base or the exponent is a variable, we need to be given additional information in order to make a numerical evaluation.

One type of exponent that was not used in the previous list of exponential expressions was a negative exponent. When dealing with a negative exponent, we have a rule to follow. The rule says:

$$b^{-x} = \frac{1}{b^x}$$

In other words, when there is a negative exponent, we need to create a fraction and put the exponential expression in the denominator and make the exponent positive. For example,

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

But working with negative exponents is just rule of exponents that we need to be able to use when working with exponential expressions.

Rules of Exponents:

$$a^m \bullet a^n = a^{m+n}$$

If the bases of the exponential expressions that are multiplied are the same, then you can combine them into one expression by adding the exponents.

This makes sense when you look at

$$2^3 \bullet 2^4 = (2 \bullet 2 \bullet 2) \bullet (2 \bullet 2 \bullet 2 \bullet 2) = 2^7$$

$$\frac{a^m}{a^n} = a^{m-n}$$

If the bases of the exponential expressions that are divided are the same, then you can combine them into one expression by subtracting the exponents.

This makes sense when you look at

$$\frac{x^7}{x^3} = \frac{x \bullet x \bullet x \bullet x \bullet \cancel{x} \bullet \cancel{x} \bullet \cancel{x}}{\cancel{x} \bullet \cancel{x} \bullet \cancel{x}} = x \bullet x \bullet x \bullet x = x^4$$

$$(a^m)^n = a^{m \bullet n}$$

When you have an exponential expression raised to a power, you have to multiply the two exponents.

This makes sense when you look at

$$(3^2)^3 = 3^2 \bullet 3^2 \bullet 3^2 = 3^{2+2+2} = 3^6$$

Notice that we had to use another rule of exponents to help us make sense of this rule. This is a common occurrence. Many times you will use more than one rule of exponents when working problems.

$$a^0 = 1$$

Any number or variable raised to the zero power is always equal to 1.

$$a^{-m} = \frac{1}{a^m}$$

This is the rule used earlier dealing with negative exponents. It is important to note that if a negative exponents already appears in the denominator of a fraction, then it will move to the numerator as a positive exponent. In short, a negative exponent changes the location (numerator or denominator) of an expression and changes the sign of the exponent. This is seen in Example 2 below.

These last two rules are not as easy to see with examples as the first three. However, they are very important to understand and are used often.

These rules are used in almost aspects of exponential expressions such as: scientific notation (link to exponents-scientific.doc), solving equations (link to exponents-solving.doc), and graphing (link to exponents-graphing.doc). So before trying any of those lessons, you should make sure you understand the examples and practice problems.

Let's Practice:

i. $(2a^{12}b^3)(3a^2b^4) =$

Everything in this problem is multiplied. The base of *a* is common so we add the exponents of 12 and 2 to get 14. The base of *b* is common so we add the exponents of 3 and 4 to get 7. The coefficients of 2 and 3 do not have any exponents to worry about and we just multiply them as they are.

$$(2a^{12}b^3)(3a^2b^4) = 6a^{12+2}b^{3+4} = 6a^{14}b^7$$

ii. $\left(\frac{3x^4y^{-3}z^2}{4x^{-3}y^{10}z}\right)^2 =$

Using order of operations tells us that we should do what is inside the parentheses first and then deal with the exponent. To simplify within the parentheses involves working with several rules including the rule for negative exponents.

$$\left(\frac{3x^4y^{-3}z^2}{4x^{-3}y^{10}z}\right)^2 = \left(\frac{3x^4z^2\ \ x^3}{4y^{10}z\ \ y^3}\right)^2$$

This step shows that the negative exponents were moved and exponents became positive.

$$\left(\frac{3x^4z^2\ \ x^3}{4y^{10}z\ \ y^3}\right)^2 = \left(\frac{3x^{4+3}z^{2-1}}{4y^{10+3}}\right)^2$$

This step shows combining exponents for terms that have the same base. Two different rules were used in this step: both the multiplication rule and the division rule.

$$\left(\frac{3x^{4+3}z^{2-1}}{4y^{10+3}}\right)^2 = \left(\frac{3x^7z}{4y^{13}}\right)^2$$

This step is the final simplification of what is inside the parentheses. Now we have to raise each term in the parentheses to the power of 2.

$$\left(\frac{3x^7z}{4y^{13}}\right)^2 = \frac{(3)^2(x^7)^2(z)^2}{(4)^2(y^{13})^2}$$

It is not absolutely necessary to use this many parentheses, but it is useful in keeping track of each term that needs to be raised to the power of 2.

$$\frac{(3)^2(x^7)^2(z)^2}{(4)^2(y^{13})^2} = \frac{9x^{14}z^2}{16y^{26}}$$

The final step is to simplify each term that has been raised to the 2nd power. It requires using the power rule for exponents.

This is a very common simplification problem. It makes extensive use of all the rules of exponents and requires several steps to get to the final answer.

Examples

$4^5 =$

What is your answer?

$\dfrac{2^{-4}}{5} =$

What is your answer?

$b^{10} \bullet b^{22} =$

What is your answer?

$\dfrac{c^{12}}{c^{10}} =$

What is your answer?

$3(a^2b^3)^2 =$

Hint

What is your answer?

$5x(4xy^2z^{-3})^3 =$

Hint

What is your answer?

$\dfrac{12d^2e^9f^{-3}}{4d^{-3}e^2f} =$

What is your answer?

$\left(\dfrac{a^{-2}}{b^3}\right)^4 \left(\dfrac{b^{-5}}{a}\right)^2 =$

Hint

What is your answer?

Check Your Answers

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