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Inside Moves

A new look at the mathematical problem of turning a sphere inside out

Turning an inflated beach ball inside out means releasing the air, reversing the ball's surface by pulling it through the opening, pumping in more air and finally resealing the ball. But a mathematical sphere has no orifice. Turning it inside out, without tearing or cutting it open, seems intuitively impossible.

Mathematicians, however, play such mental games by their own rules. If it were possible to push a surface through itself, meaning that two points on the surface could temporarily occupy the same point in space, then a solution might exist to the problem of reversing, or everting, a sphere's surface. The trick is performing the entire eversion without letting a crease enter the picture.

Thirty years ago, Stephen Smale, now at the University of California, Berkeley, proved it is possible to turn a sphere inside out, but he provided no simple way of visualizing the procedure. A number of mathematicians have since pictured the transformation, and they continue to look for simpler, more efficient means of describing and displaying how the change occurs.

"It's a problem that captures the imagination," says mathematician Anthony Phillips of the State University of New York at Stony Brook. "It's a fairly easy problem to explain. It's something that somehow should be very simple to do but is really very complicated."

Now French mathematician Bernard Morin of the Université Louis-Pasteur in Strasbourg has found the simplest possible route for a sphere eversion. "I just need to follow the positions of 12 points," Morin says. The coordinates of those points throughout the transformation provide all the information a mathematician needs to understand the eversion.

This exercise falls within the mathematical field known as topology, which concerns the fundamental properties of shapes. By studying the steps required to transform one shape into another, topologists establish relationships between different shapes and learn the key differences distinguishing one shape from another. Such abstract notions play an increasingly important role in many disciplines outside of mathematics, from molecular biology (SN: 11/12/88, p.319) to particle physics and cosmology (SN: 3/18/89, p.174).

Intuition suggests that the mathematical problem of everting a sphere without allowing creases can't be solved. Imagine a sphere painted blue on the outside and red on the inside. Pushing the north and south poles toward the sphere's center, then past each other, forces the original inner surface to protrude more and more. The transformed object begins to look like a red sphere with a blue tube running around its equator. Gradually the blue tube (the remaining portion of the outside) becomes thinner and thinner until it vanishes. That leaves a sphere with a red outside and a tight loop that must be pulled through itself. That produces a sharp crease, which isn't allowed.

In 1959, Smale, then a graduate student, proved an abstract theorem that indirectly leads to the proposition making sphere eversions possible. The result so surprised his thesis adviser, Raoul H. Bott, now at Harvard University, that Bott insisted Smale had to have made a mistake.

But the logic of Smale's proof held up. In fact, his proof laid out a step-by-step path for accomplishing a sphere eversion, but in such a complicated argument that no one could visualize his procedure. Thus, for some time after Smale's discovery, mathematicians knew that turning a sphere inside out was possible, yet no one had the slightest idea how to do it.

Eventually, a number of mathematicians, including Phillips and Morin, devised workable sphere-eversion schemes involving complex sets of moves that stretch, pinch and twist the surface through the crucial stages during the transformation. Morin's latest effort reduces the problem to following the coordinates of 12 points on a sphere -- making a sphere, for the purposes of this problem, equivalent to a polyhedron with 12 corners, or vertices.

Morin starts with a cuboctahedron, which looks like a cube with its corners lopped off. This polyhedron has 12 vertices and 14 faces (six squares and eight equilateral triangles). By using a sequence of elementary moves (moving a vertex along an edge), Morin transforms the cuboctahedron into a curiously shaped figure, which he calls the "central model," with only 12 faces but the same number of vertices as before. Four of its faces are nonconvex pentagons, which look like notched quadrilaterals. The rest are triangles.

A sequence of six elementary moves carries the central model through the tricky stages of the eversion. A final flurry of moves produces an octahedron again, now turned inside out.

Morin's achievement is remarkable on several counts. In his history of sphere eversions in A Topological Picturebook (Springer-Verlag, New York, 1987), mathematician George K. Francis of the University of Illinois at Urbana-Champaign writes: "Bernard Morin is not distracted, like the rest of us, by pencil and paper and the business of drawing and looking at pictures. He is blind. With superb spatial imagination, he assembles complicated homotopies [transformations] of surfaces directly in space. He keeps track of temporal changes in the double curves and the surface patches spanning them. His instructions to the artist consist of a vivid description of the model in his mind."

Morin's polyhedral model of a sphere eversion makes it easier to keep track of where all the pieces of a surface are going during a transformation. "This way, you can intellectually throw away a lot of the complications and just focus on the essential parts," Phillips says.

"If you start with the 12 vertices of a cuboctahedron, you see many phenomena," Morin adds. "You are forced to see the twists. You see that you have to contract enormously certain things and elongate enormously other things."

Although Morin's version is the simplest possible polyhedral model of a sphere eversion, it isn't the first. Last year, John F. Hughes of Brown University in Providence, R.I., created a polyhedral model with a larger number of vertices in an effort to find a set of equations he could use to program a computer to perform and display a complete sphere eversion.

Hughes' approach to finding an explicit formula was to build up a given surface from small patches already defined by specific mathematical expressions. It's a process akin to sewing together a patchwork quilt. Mathematically, the idea is to piece together algebraic expressions known as polynomial functions, each defining a small piece of surface, adding them together and making sure the patches meet smoothly.

Hughes has used his equations to produce an animated film that vividly demonstrates a sphere eversion. He's now working on an alternative model in which all the steps of a sphere eversion appear as different slices through a particular surface in four-dimensional space.

"In some sense, because it's an attempt to visualize a fairly abstract thing, the sphere-eversion problem is a never-ending quest," Francis says. "There's always something left to be done in the visualization part."

PHOTO (BLACK & WHITE): This computer-generated picture, constructed from about 12,000 polygons, represents one stage in the polyhedral sphere eversion worked out by mathematician John

Hughes.

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By Ivars Peterson
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