

## Perplexing Task 2

Create a triangle and label your vertices A, B, and C. Using any tool you wish, construct the orthocenter of your triangle and label it H. Investigate the following:

1. Make a triangle using any three of the four points A, B, C, and H. Find the orthocenter of your new triangle. What happened? Why do you think this happened?
2. Reflect H over each of the sides of triangle ABC. What appears to be true about the three reflections of H? Why do you think this happens?
3. Make two different triangles using any three of the four points A, B, C, and H. Construct the circumcircle of your triangles. What appears to be true about the circles? Why do you think this happens?

A theorem is a statement that can be proven using definitions, postulates, and other theorems. Generalize your findings in parts 1, 2, and 3 to write your own theorems.

You can use any means of construction to investigate these ideas.

In part 1, no matter which three points the participant uses to form a triangle, the fourth point will be the orthocenter of that triangle. Since the orthocenter is the point of intersection of the altitudes, the perpendicular relationships will remain in all of the triangles.

In part 2, the six points A, B, C, and all three of the reflections of H, lie on the circumcircle. There are other truths that the participants may observe. Encourage as many true observations as possible. When statements are made that are untrue, it is often the case that participants have not checked to be sure that the statements are *always* true. If participants struggle too much with this part of the problem, encourage them to move on to part 3 and come back to 2 for more work. Part 3 may give them more insight into part 2.

In part 3, the circumcircles of all four possible triangles are congruent. The question does not direct participants to check all four. Hopefully, as they begin to generalize, they will realize that in each part of this problem they need to check several possibilities in order to test their conjectures. It is probably best to observe as they do parts 1, 2, and 3, without many comments or questions, as long as they are not completely wrong. Then start asking more guiding questions, as needed, when they begin to write their theorems.