Function Learning Task (Function Notation)

1. One vacation when visiting his grandmother, Todd found markings on the inside of a closet door showing the heights of his mother, Julia, and her brothers and sisters on their birthdays growing up. From the markings in the closet, Todd wrote down his mother's height from ages 2 to 16. His grandmother found the measurements at birth and one year by looking in his mother's baby book. The data is provided in the table below, with heights rounded to inches.

Age (yrs.)	x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Height (in.)	$y = h(x)$	21	30	35	39	43	46	48	51	53	55	59	62	64	65	65	66	66

   a. Which variable is the independent variable, and which is the dependent variable? Explain your choice.
   b. Make a graph of the data.
   c. Should you connect the dots on your graph? Explain.
   d. Describe how Julia's height changed as she grew up.
   e. How tall was Julia on her 11th birthday? Explain how you can see this in both the graph and the table.
   f. What do you think happened to Julia's height after age 16? Explain. How could you show this on your graph?

2. In Math 1, we will often use *function notation* to describe relationships between quantities that vary. In function notation, $h(2)$ means the output value when the input value is 2. In the case of the table above, $h(2)$ means the y-value when x is 2, which is her height (in inches) at age 2, or 35. Thus, $h(2) = 35$. Function notation gives us another way to write about ideas that you began learning in middle school, as shown in the table below.

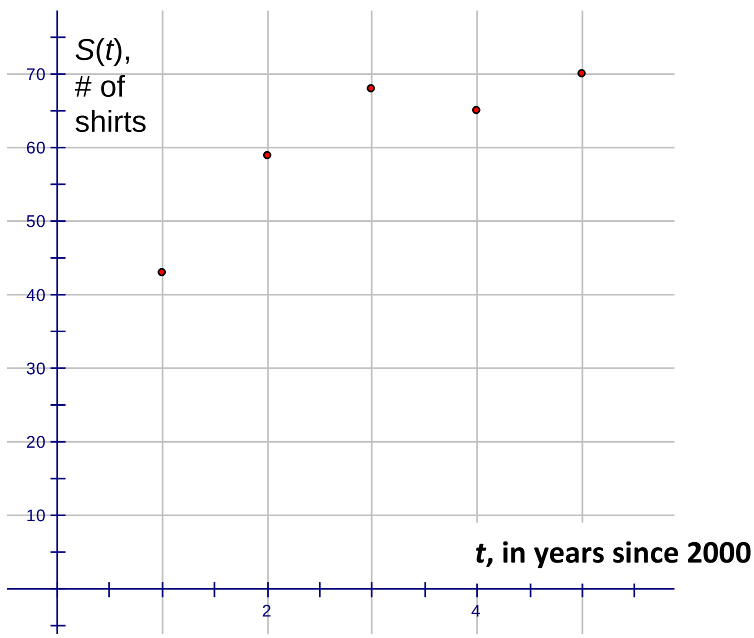
Statement	*Type*
At age 2, she was 35 inches tall.	Natural language
When x is 2, y is 35.	Statement about variables
When the input is 2, the output is 35.	Input-output statement
$h(2) = 35$.	Function notation

As you can see, function notation provides shorthand for talking about relationships between variables. With function notation, it is easy to indicate simultaneously the values of both the independent and dependent variables. The notation $h(x)$ is typically read "h of x," though it is helpful to think "h at x," so that $h(2)$ can be interpreted as "height at age 2," for example.

Note: Function notation looks like an instruction to multiply, but the meaning is very different. To avoid misinterpretation, be sure you know which letters represent functions. For example, if *g* represents a function, then *g*(4) is *not* multiplication but rather the value of "*g* at 4," i.e., the output value of the function *g* when the input is value is 4.

a. What is *h*(11)? What does this mean?
b. When *x* is 3, what is *y*? Express this fact using function notation.
c. Find an *x* so that *h*(*x*) = 53. Explain your method. What does your answer mean?
d. From your graph or your table, estimate *h*(6.5). Explain your method. What does your answer mean?
e. Estimate an *x* so that *h*(*x*) = 60. Explain your method. What does your answer mean?
f. Describe what happens to *h*(*x*) as *x* increases from 0 to 16.
g. What can you say about *h*(*x*) for *x* greater than 16?
h. Describe the similarities and differences you see between these questions and the questions in the previous problem.

3. When Peachtree Plains High School opened in 2001, a few teachers and students put on FallFest, featuring contests, games, prizes, and performances by student bands. To raise money for the event, they sold FallFest T-shirts. The event was very well received, and so FallFest has become a tradition. The graph below shows T-shirt sales for each FallFest so far.



a. What are the independent and dependent variables shown in the graph?
b. For which years does the graph provide data?
c. Does it make sense to connect the dots in the graph? Explain.
d. What were the T-shirt sales in 2001? Use function notation to express your result.
e. Find $S(3)$, if possible, and explain what it means or would mean.
f. Find $S(6)$, if possible, and explain what it means or would mean.
g. Find $S(2.4)$, if possible, and explain what it means or would mean.
h. If possible, find a t such that $S(t) = 65$. Explain.
i. If possible, find a t such that $S(t) = 62$. Explain.
j. Describe what happens to $S(t)$ as t increases, beginning at $t = 1$.
k. What can you say about $S(t)$ for values of t greater than 6?

Note: As you have seen above, functions can be described by tables and by graphs. In high school mathematics, functions are often given by formulas, but it is important to remember that not all functions can be described by formulas.

4. Suppose a ball is dropped from a high place, such as the Tower of Pisa. If y, measured in meters, is the distance the ball has fallen and x, measured in seconds, is the time since the ball was dropped, then y is a function of x, and the relationship can be approximated by the formula $y = f(x) = 5x^2$.

x	0	1	2	3	4	5	6	…
x^2	0	1	4	9				…
$y = f(x) = 5x^2$	0	5	20					…

a. Fill in the missing values in the table above.
b. Suppose the ball is dropped from a building at least 100 meters high. Measuring from the top of the building, draw a picture indicating the position of the ball at times indicated in your table of values.
c. Draw a graph of x versus y for this situation. Should you connect the dots? Explain.
d. What is the relationship between the picture (part b) and the graph (part d)?
e. You know from experience that the speed of the ball increases as it falls. How can you "see" the increasing speed in your table? How can you "see" the increasing speed in your picture?
f. What is $f(4)$? What does this mean?
g. Estimate x such that $f(x) = 50$. Explain your method. What does it mean?
h. In this context, y is proportional to x^2. Explain what that means. How can you see this in the table?

5. Towanda is paid \$7 per hour in her part-time job at the local Dairy Stop. Let t be the amount time that she works, in hours, during the week, and let $P(t)$ be her gross pay (before taxes), in dollars, for the week.
    a. Make a table showing how her gross pay depends upon the amount of time she works during the week.
    b. Make a graph illustrating how her gross pay depends upon the amount of time that she works. Should you connect the dots? Explain.
    c. Write a formula showing how her gross pay depends upon the amount of time that she works.
    d. What is $P(9)$? What does it mean? Explain how you can use the graph, the table, and the formula to compute $P(9)$.
    e. If Towanda works 11 hours and 15 minutes, what will her gross pay be? Show how you know. Express the result using function notation.
    f. If Towanda works 4 hours and 50 minutes, what will her gross pay be? Show how you know. Express the result using function notation.
    g. One week Towanda's gross pay was \$42. How many hours did she work? Show how you know.
    h. Another week Towanda's gross pay was \$57.19. How many hours did she work? Show how you know.